

BEHAVIOR OF CONCRETE UNDER DYNAMIC COMPRESSIVE LOADING

BY

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL  
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1990

## ACKNOWLEDGMENTS

The author is indebted to Professor Lawrence E. Malvern for his thoughtful guidance through the fulfillment of this work. To the author, he has not only granted invaluable academic advice in the research but also provided an example of responsibility and dedication.

Appreciations are extended to Professor Daniel C. Drucker, Dr. David A. Jenkins, Professor John M. Lybas and Professor Edward K. Walsh for their serving on the author's supervisory committee, especially to Professor Daniel C. Drucker for his kindly reviewing the constitutive modeling as well as his interesting course of Plasticity, and especially to Dr. David A. Jenkins for his tremendous precious help in the experimental work.

The author is indebted to his wife Yaoqi, his parents, his brothers and his son. Without their inspiration, encouragement and forbearance, this work would be impossible.

Finally, appreciations are extended to the support by the United States Air Force Office of Scientific Research, under Contract No. AFOSR F49620-83-K-0007 and Contract No. AFOSR-87-0201.

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Abstract of Dissertation Presented to the Graduate School  
of the University of Florida in Partial Fulfillment of  
the Requirements for the Degree of Doctor of Philosophy

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by

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May 1990

Chairman: Lawrence E. Malvern

Major Department: Aerospace Engineering, Mechanics and  
Engineering Science

This research is to study behavior of concrete under dynamic compressive loading. Unconfined high-strength concrete specimens were uniaxially tested on a three-inch-diameter Split Hopkinson Pressure Bar. From the experimental data, a rate-dependent model for the uniaxial unconfined compression case was proposed, which included inelastic-strain-rate dependence of stress up to failure and a failure-initiation criterion of an accumulative type. Furthermore, an overstress-type elastoviscoplastic constitutive model for the three-dimensional stress case was constructed to combine a proposed static elastoplastic cap model for concrete with the characteristics of concrete

under dynamic loading, which were observed by the dynamic tests of this research.

The proposed elastoviscoplastic model was then implemented in a two-dimensional time-varying finite element program, which is applicable to concrete and concrete structures under impulsive loading. As a computation example, the program was applied to the geometry of the concrete specimen so that computation results were compared with experimental data to validate the accuracy of the constitutive model.

In addition to the extensive experimental investigation of high-strength concretes and the modeling and computational studies, a preliminary experimental investigation was made of the crack accumulation in smaller specimens of a concrete of moderate strength after various amounts of deformation. This was accomplished by interrupting the tests after various amounts of deformation, so that micrographic examinations could be performed on sections cut from the intact recovered specimens.

## CHAPTER 1 INTRODUCTION

Although concrete has been used more than any other engineering material and many concrete structures have to carry impulsive loads now and then, very little knowledge about dynamic response of concrete has been acquired. Some early research indicated rate effects on concrete strength, but systematic experimental data and theoretical interpretation have been lacking. As a result, regardless of the great increase in development and application of many modern computer-based techniques such as the finite element, finite difference, and boundary integral equation methods, numerical analysis for structural response of reinforced concrete used as a parameter the unconfined compressive strength of concrete,  $f_c$ , which was sometimes increased by an arbitrary factor for dynamic loading. The uncertainty of the results from numerical analysis on the concrete structure under impulsive loading has spurred active research and interest in the dynamic experiments and rate-dependent constitutive theories of concrete.

A series of research studies at the University of Florida on dynamic properties of concrete started with



building a 3-in (76.2-mm) diameter Kolsky apparatus or Split Hopkinson's Pressure Bar (SHPB) system in 1984 at the Aerospace Engineering, Mechanics and Engineering Science Department (then called the Engineering Sciences Department), because one of the most important reasons for the shortage of credible experimental data for dynamic response of concrete was that all the testing devices ever used before, including the falling hammer and Charpy pendulum, were not reliable to determine stress and strain histories of a specimen. Most of the existing SHPB systems have a maximum specimen diameter of 1 in (25.4 mm), but in structures of interest concrete aggregate sizes up to 0.5 in (12.5 mm) or even larger may be used. For such concrete, a larger specimen must be used in order to obtain representative properties of the composite material. The 3-in-diameter system was developed for this reason.

With this system, unconfined 3-in-diameter specimens of five kinds of high-strength concrete (all with nominal static compressive strength of 14 ksi) were tested and each of them showed strain-rate dependence of dynamic compressive strength. Mortar specimens were tested with a small SHPB system of 3/4-in-diameter before the 3-in-diameter system had been set up. The mortar also showed strength enhancement by the strain rate. Strain gages were directly mounted on the lateral surface of some 3-in-diameter concrete specimens of two kinds of high-strength concrete.

Analysis of all the information provided by the tests of the two kinds of high-strength concrete suggested a uniaxial rate-dependent model of concrete, which included a law of strain-rate dependence of stress up to the maximum stress, and a cumulative criterion for occurrence of the maximum stress or failure initiation. This model satisfactorily describes the deformation of concrete under uniaxial dynamic compressive loading up to the failure point.

In addition, behavior of concrete in the three-dimensional stress case was studied. An elastoplastic model for concrete was proposed, which was a modification of existing models. It consists of a stationary failure surface and a cap-shaped strain-hardening surface in the principal-stress space. By combining this model with the characteristics of response of concrete under dynamic loading, which were disclosed in the dynamic tests, an overstress type of elastoviscoplastic model was proposed.

The proposed elastoviscoplastic model was implemented in a two-dimensional time-varying finite element program, which is applicable to concrete structures in the plane stress, plane strain or axisymmetric cases. The program was applied to the concrete specimen, and output data from the computation coincided well with the experimental data.

Although some mathematical descriptions originally proposed for the behavior of metals such as plasticity and

viscoplasticity can be used for concrete under some conditions, the actual physical deformation processes are quite different in concrete. Concrete undergoes a process of crack initiation, propagation and coalescence from deformation through fracture. In a preliminary experimental investigation, a technique was devised for stopping the SHPB tests after various amounts of deformation and recovering intact specimens for micrographic examination. The investigation demonstrated that the technique would work and that slices cut from the recovered specimens could be stabilized to prevent further damage during grinding and polishing before micrographic examination. A more extensive investigation is needed to provide evidence that might be used to verify micromechanically-based models of the deformation and failure.

This dissertation consists of eight chapters. Following this chapter, in addition to a background literature survey of dynamic tests on concrete, Chapter 2 describes the SHPB system and refers to the theories applied to the SHPB system, including the theories for wave dispersion correction, and test-data treatment. In Chapters 3 and 4, test data are analyzed to present behavior of unconfined concrete specimens under dynamic compressive loading. Lateral inertia analysis and strain-rate dependence of the strength are given in Chapter 3, while the uniaxial rate-dependent model is included in Chapter 4.

Chapter 5 covers the experimental observation of crack damage. Chapter 6 furnishes the three-dimensional elastoplastic and elastoviscoplastic modeling, and Chapter 7 presents the computation algorithm of an existing finite element program. A background survey of plastic and viscoplastic theories for concrete is included in Chapter 6. Computation examples on the concrete specimen are given in Chapter 7, where effects of friction between the specimen and the bars of the SHPB system are discussed. Finally, Chapter 8 summarizes conclusions and proposes some recommendations for further study.

## CHAPTER 2 THE SHPB SYSTEM AND DYNAMIC TESTS

### 2.1 Background

Although concrete is the material which is used more than any other engineering material in quantity over the world every year, and numerous engineering structures made mainly of concrete are to carry impulsive loads or their boundaries possibly encounter transient displacements, the dynamic response of concrete to impulsive loads is not well understood. Malvern et al. (1986b, 1989a) summarized the previous experimental investigations on properties of concrete under dynamic loading. Their summaries are adapted as follows.

Early work on rate effects in concrete was reviewed in a 1956 ASTM symposium (McHenry and Shideler, 1956). At stress loading rates from 1 to 1000 psi per second in testing machines, compressive strength was reported to be a logarithmic function of the loading rate with recorded strength increases up to 109% of the strength reported at "standard rates" (20 to 50 psi per sec). By using cushioned impact tests Watstein (1953) obtained strengths 185% of the standard. Watstein also reported rate effects on the

measured secant modulus of elasticity, energy absorption (up to 2.2 times the static value) and strain to failure.

Dynamic elastic properties of concrete bar specimens have been studied by Goldsmith and his colleagues (1966, 1968) by analyzing longitudinal wave propagation induced by impacting the end of a long bar with a steel sphere. Dynamic tensile strengths have been measured in similar longitudinal wave impacts with small projectiles by Birkimer and Lindemann (1971) and at the University of Florida by Griner (1974) and by Sierakowski et al. (1977).

Read and Maiden (1971) surveyed the state of knowledge on the dynamic behavior of concrete at high stress levels. At the higher stress levels the most extensive experimental data then available were those of Gregson (1971), who used a gas gun to provide flyer plate impacts against thin concrete plates and determine the uniaxial strain response at pressures from 40 to 8,000 ksi.

Watstein's "cushioned impact" tests used a 140-lb hammer striking a specimen set in plaster of paris atop a dynamometer mounted on a 3200-lb anvil (1953). The specimen was capped by a steel plate. Green (1964) and Hughes and Gregory (1968) used a ballistic pendulum with a 25-lb hammer having a one-inch-diameter face, which impacted one face of a 4-inch cube specimen mounted against an anvil. The anvil was also suspended as a pendulum to measure retained kinetic energy. Atchley and Furr (1967) used a drop tester with

drop heights up to 20 ft and found dynamic strengths up to more than 1.6 times the static strengths. Seabold (1970) represented the effects on unconfined compressive strength in concrete up to strain rates of about 5/sec by an empirical formula containing both a linear term and a logarithmic term in the strain rate.

Hughes and Gregory (1972) have used a drop tester with a 48.2-lb hammer falling through a height of up to more than 6 feet. The specimens were mainly 4-in cubes, although some 4x4x8 inch prisms were tested. Specimens were fitted with metal plates on top and bottom and mounted atop a steel load column fitted with strain gages to record the transmitted force. The load column was not of uniform diameter, and it was too short to avoid reflected waves during the recording, but by using a simple bar-wave theory they estimated transient stresses and concluded that the impact strengths averaged about 1.92 times the static compressive strength. This is comparable to the dynamic strength increases reported by Watstein in 1953. For some of their "strong concretes" dynamic compressive strengths above 260 MPa (37.7 ksi) were found by Hughes and Gregory (1972). Hughes and Watson (1978) used a similar test technique and concluded that the percentage increase in compressive strength in dynamic tests over the static strength was greater for low-strength concrete than for high-strength concrete.

SHPB systems have been used to determine the properties of geotechnical materials and concrete. Lundberg (1976) investigated energy absorption in dynamic fragmentation of Bohus granite and Solenhofen limestone specimens of 25-mm diameter and 50-mm length in a compressive system at striker bar speeds from 2.5 to 20 m/s. Specimen strain rates were not reported, but general fracture of the specimens occurred when the stress was about 1.8 times the static strength for Bohus granite and 1.3 times for Solenhofen limestone.

Bhargava and Rehnström (1977) found unconfined dynamic compressive strengths of 1.46 to 1.67 times the static strength in plain concrete and fiber-reinforced and polymer-modified concrete. Their failure strengths were identified as the maximum amplitude of short-pulse stresses that could be transmitted through the specimen and were more associated with the onset of failure than with complete crushing of the 100-mm diameter by 200-mm length cylinders. Their vertical bar apparatus was 36 meters high, and the 250-mm-long striker bar could be dropped through heights up to 10 meters to impact the 2-meter-long input bar. Their short striker bar gave a loading pulse duration only about twice the transit time through an elastic specimen.

At the University of Florida, Sierakowski et al. (1981) found dynamic compressive strengths around 26.4 ksi in 3/4-inch-diameter SHPB specimens of fine-grained concrete. This was 1.93 times the static compressive strength of 13.7 ksi.



These specimens had been aged four years in laboratory storage in a dry place. They were cut from some of the same bars previously tested by Sierakowski et al. (1977), who had found dynamic strengths around 9.0 ksi as compared with static strengths of 7.9 ksi for 28-day concrete. The static strengths of the four-year concrete were measured on small specimens (3/4-inch-diameter). Few other applications of SHPB technology to concrete had been made before the research at the University of Florida. Kormeling et al. (1980) adapted the SHPB for dynamic tensile tests. They reported dynamic tensile strengths of more than twice the static value at strain rates of approximately 0.75/sec.

Suaris and Shah (1982) have surveyed mechanical properties of materials subject to impact and rate effects in fiber-reinforced concrete. Many investigators have found significant increases in impact tensile strength in fiber-reinforced concrete over that of plain concrete (Hoff, 1979).

Radial inertia effects in geotechnical materials, which may be mistaken for strain-rate effects, have been discussed by Glenn and Janach (1977). They observed high radial accelerations, which they attributed to dilatancy in their granite specimens, and suggested that the lateral confinement induced by this lateral inertia causes the core of the cylinder to be more nearly in a state of uniaxial strain than uniaxial stress. Their tests involved direct

impact of the striker against the specimen at such high speeds that failure occurred during the first transit of the wave through the specimen.

Young and Powell (1979) investigated lateral inertia effects in SHPB tests on Solenhofen limestone and Westerly granite. By means of embedded wire loops and an externally applied longitudinal magnetic field they measured radial velocities, and hence determined radial accelerations and calculated radial-stress confinement in tests with loading pulse lengths of 50 microseconds in specimens of length 46 to 52 mm. The impact speeds used (29.9 to 61.4 m/s) apparently caused failure during the first passage of the stress wave through the specimen. They concluded that the induced radial stress field is capable of producing a confining pressure sufficient to account for increases of compressive strength observed in SHPB experiments.

Bertholf and Karnes (1975) made a two-dimensional numerical analysis of the SHPB system for testing metal specimens and concluded that with lubricated interfaces the one-dimensional elastic-plastic analysis was reasonable if limitations are imposed on strain rate and rise time in the input pulse and if specimen length to diameter is about 0.5.

More relevant to localized impact loadings would be dynamic tests of concrete cylinders with various degrees of lateral confinement. Dynamic tests of concrete specimens with a controlled lateral confinement are extremely rare. A

few tests have been reported at varying strain rates and confining pressures in a testing machine.

Takeda et al. (1984) have performed triaxial tests on concrete cylinders loaded by compressive and tensile axial loads with axial straining rates of  $2 \times 10^{-3}/\text{sec}$ ,  $2 \times 10^{-2}/\text{sec}$  and  $2/\text{sec}$  after statically applied confining pressure up to 1.5 times the uniaxial compressive strength had been initially imposed. Plots of their results in the form of octahedral shear stress versus octahedral normal stress, both normalized by dividing by the uniaxial compressive strength at the particular rate of straining, showed no significant differences among the relations at the three rates of strain. Even unconfined dynamic compressive strength had not previously been established for strain rates above  $10/\text{sec}$  because of uncertainty about the actual stress levels in the impact tests.

Recently Yamaguchi and Fujimoto (1989) reported a similar triaxial testing program at axial compressive strain rates of order  $10^{-6}$  to  $10^{-1}/\text{sec}$ . For unconfined uniaxial tests over this range of rates the maximum stress was increased by as much as 50 percent. In the triaxial tests, the lateral pressure supplied by the hydraulic pump could not be held constant during the high-rate axial compression but decreased during the test. The result was that the initial parts of the higher-rate axial stress-strain curves increased with strain rate (as did the initial modulus), but

at the higher axial strains the increase was smaller and in many cases the high-rate curves even fell below the static curve at the same initial lateral pressure, because the lateral pressure was constant in the static test.

Gran et al. (1989) devised a method to study the tensile failure of brittle geologic materials at strain rates from 10 to 20/sec. A cylindrical rod was first loaded in triaxial compression. Then the axial load was released very rapidly, allowing rarefaction waves to propagate along the rod to interact at the center to form a tensile stress equal in magnitude to the initial static axial compression. They estimated that, for their concrete specimens, at axial strain rates of 10 to 20/sec the tensile strength with 10 MPa (1450 psi) lateral confining pressure was about 100% higher than the unconfined static splitting tensile strength and about 40% higher than the unconfined dynamic tensile strengths at the same rates.

Dynamic triaxial tests of high strength concrete were reported by Gran et al. (1989) with both axial and lateral pressure applied by the venting of explosive gases. Their axial compressive strain rates varied from 0.5 to 10/sec, and lateral confining pressures from zero to 124 MPa (18 ksi). Little rate effect was noted at the lower rates, but at an axial compressive strain rate of 6/sec, the elastic modulus was about 60% higher and the unconfined compressive strength about 100% higher than the static values (at

$10^{-4}$ /sec). The triaxial compressive data established an approximate shear failure envelope, for strain rates between 1.3 and 5/sec, which was 30 to 40 percent higher than that for static loading.

Christensen et al. (1972) used a compressive SHPB system to test nugget sandstone specimens under confining pressure up to 207 MPa (30 ksi). They also used truncated cone striker bars to spread out the rise time of the loading pulse in order to improve the accuracy and resolution of the initial portion of the stress-strain curves. Lindholm et al. (1974) tested specimens of Dresser basalt under confining pressure up to 690 MPa (100 ksi) at strain rates up to  $10^3$ /sec and considered also temperature. They found unconfined strength variation by a factor of 3 over the range of rate and temperature examined. The dependence of the ultimate strength on rate and temperature indicated that fracture was controlled by a thermal activation process. All the experimental data correlated well with a proposed thermal activation fracture criterion.

Some SHPB tests on concrete specimens passively confined by close-fitting steel or aluminum jackets were performed at the University of Florida (Gong, 1988). The confining pressure developed by the jacket in constraining the specimen expansion was estimated from strain-gaged measurements of hoop strains on the outer wall of the jacket, assuming elastic response of the jacket. The

pressure was not controllable, but increased during each test from zero to a maximum of from about 22 MPa (3.2 ksi) to 44 MPa (6.4 ksi) depending on the impact speed of the SHPB striker bar.

More recently, a hydraulic pressure cell was designed and installed on the SHPB system at the University of Florida to test plain concrete specimens in axial dynamic compression while they were subject to moderate lateral confining pressure (Malvern et al., 1989b). The cell was used in dynamic tests at initial lateral confining pressures from 485 to 1500 psi (3.3 to 10.3 MPa) on concrete specimens. The confined axial dynamic stress-strain responses differed markedly from the unconfined response. The test results showed a significant increase of strength with confining pressure as well as rate sensitivity of strength.

More efforts have probably been made to study dynamic properties of metals in the recent decades. Both the experimental and the theoretical approaches to the strain-rate dependent behavior of metals are more mature than those of concrete and geological materials. Though concrete changes its structure in deformation in a way different from metals, much in both experimental methods and mathematical constitutive theories originally developed for metals can be used to help the study of concrete properties. Therefore, it would be helpful to review the history of the

establishment of the strain-rate dependence theory of metals (For more detailed history, see Nicholas, 1982). The problem was raised in the early 1950s whether the strain rate should be taken into the constitutive equation in analyzing a longitudinal elastoplastic stress wave in a bar. The first plausible one-dimensional, finite-amplitude plastic wave propagation theory was developed independently by von Karman in 1942 in the United States, G. I. Taylor in 1942 in England and Rakhmatulin in 1945 in Russia. The theory assumed that the behavior of a material could be described by a single-valued relation between stress and strain in uniaxial stress, which has nothing to do with the strain rate. However, their theoretical solutions deviated from some of the experimental data. Sokolovskii in 1948 and Malvern in 1951 (Malvern, 1951) independently suggested a strain-rate dependent theory in the same form, which well interpreted the experimental data from the bar wave. Since then, a great many experimental investigations and theoretical discussions were carried on, which created an atmosphere where dynamic experiments and viscoplasticity theories promoted each other in their developments. For the theories of viscoplasticity, Chapter 7 will present more discussion. Both the experimental investigations and the theoretical achievements for dynamic behavior of metals provide the research on concrete with a solid background.

## 2.2 The Configurations of the SHPB Systems

This research uses the SHPB (Split Hopkinson Pressure Bar) system to dynamically test concrete and mortar specimens. Since Kolsky reformed the classic Hopkinson Pressure Bar to the split form for determining material properties with high strain rates in 1949, the SHPB system or Kolsky apparatus has been used widely around the world. Many authors have described its structure and applications. See, e.g., Lindholm (1964). The system consists mainly of an incident bar and a transmitter bar, with a short specimen sandwiched between them, and a striker bar which impacts the incident bar to produce a longitudinal compressive pressure pulse. The incident compressive pulse propagates along the incident bar toward the specimen. When it reaches the specimen, part of it reflects and turns backward as a reflected pulse, which is tensile because the specimen has a lower impedance than the pressure bars, and another part of it keeps going ahead, traveling through the specimen, and part of it continues into the transmitter bar. Reflecting at the two interfaces, part of the pulse in the specimen goes back and forth and makes the stress along the specimen approximately uniform after a number of reflections. The compressive pulse that leaves the specimen and travels forward along the transmitter bar is called the transmitted pulse. The pressure bars remain elastic in the test. By



analysis of the elastic stress waves in the pressure bars, the velocities of the ends in contact with the specimen (hence the average strain rate of the specimen) and the stresses of the specimen at its two end faces are obtained from the measurements of the incident, reflected and transmitted pulses.

The Department of Aerospace Engineering, Mechanics and Engineering Science, the University of Florida, has two SHPB systems. One has the bars of 3/4 inch (19.05 mm) in diameter and the other has the bars of 3 inches (76.2 mm) in diameter. According to the different sizes of the bars, they will be referred to as the Small Bar and the Big Bar, respectively, in the following for brevity. Their specifications are listed in Table 2.1. Figure 2.1 is an overview of the Big Bar. The striker bar in the Small Bar, which was made for the University of Florida by the Southwest Research Institute, is driven by a torsion spring as described by Lindholm (1964). In the Big Bar, a gas gun propels a projectile that acts as a striker bar. The gas gun was fabricated by Terra-Tek Systems, Inc. As currently configured, it fires a 32-in-long, 3-in-diameter, 61.28-lb steel striker bar to impact at speeds  $V_0$  up to 57.5 ft/sec with firing chamber pressures up to 600 psi. Higher speeds are possible with a lighter impactor or a higher firing pressure. The propellant gas is furnished by a nitrogen bottle (2000 psi maximum). The firing chamber was proof

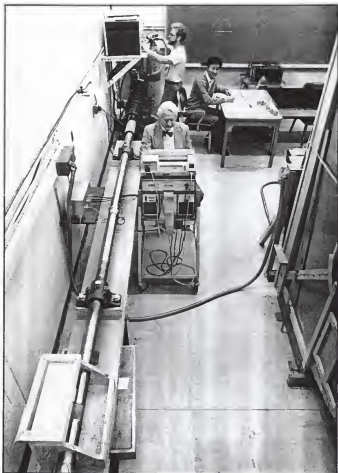


Figure 2.1 Overview of the 3-in SHPB System

(From Research Report, 1985-1986,  
Engineering & Industrial Experiment Station,  
University of Florida, Gainesville)

Table 2.1 Specifications of  
the SHPB Systems at the University of Florida

| Specification                           | 3-in system                                         | 3/4-in system                                       |
|-----------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| Diameter of bars                        | 0.0762 m                                            | 0.01905 m                                           |
| Length of the<br>striker bar            | 0.81 m<br>(32 in)                                   | 0.58 m<br>(23 in)                                   |
| Length of the<br>incident bar           | 3.10 m<br>(122 in)                                  | 1.78 m<br>(70 in)                                   |
| Length of the<br>transmitter bar        | 3.10 m<br>(122 in)                                  | 1.78 m<br>(70 in)                                   |
| Gages on incident<br>bar from specimen  | 1.52 m<br>(59.9 in)                                 | 0.88 m<br>(35 in)                                   |
| Gages on trans.<br>bar from specimen    | 1.52 m<br>(59.9 in)                                 | 0.89 m<br>(35 in)                                   |
| Bar-wave speed<br>$C_0 = \sqrt{E/\rho}$ | 5160 m/sec<br>( $2.03 \times 10^5$ in/s)            | 5210 m/sec<br>( $2.05 \times 10^5$ in/s)            |
| Density of the<br>bar material $\rho$   | 7650 kg/m <sup>3</sup><br>(7.65 g/cm <sup>3</sup> ) | 7830 kg/m <sup>3</sup><br>(7.83 g/cm <sup>3</sup> ) |
| Poisson's ratio                         | 0.32                                                | 0.28                                                |

tested to 3000 psi, but, as currently configured, a safety valve in the control system limits the firing pressure to 750 psi to avoid damaging the pressure gauge. The 3-in-diameter striker bar is guided in the 3.125-in-diameter gun barrel by two supporting sleeves.

Both the Small Bar and the Big Bar employ strain gages to measure the strain pulse along the incident bar and the transmitter bar. A full strain-gaged bridge is permanently mounted on each pressure bar at the mid-point of the bar length. Each bridge consists of two double element strain

gages (Micro-Measurements Type WA-06-250TB-350) mounted on opposite sides of the bar. The gage elements are oriented in the longitudinal and transverse directions of the bar. The amplified signals are recorded by a transient recorder. The recorder used in mortar tests was a Nicolet Model 206 digital oscilloscope, from which acquired data were transferred to a microcomputer and then stored in a floppy disk. All the concrete tests were recorded with a Nicolet 4094, which was purchased after the mortar tests were completed. The Nicolet 4094 used has four channels, and is microcomputer and disk-driver equipped. It not only displays acquired signals on the screen and stores them in diskettes, but also performs simple mathematical operations, such as addition, subtraction, multiplication, division, differentiation and integration, of the signals. In all the tests of concrete and mortar, time resolution (time division of datum points) of 0.5 microsecond is used. The Nicolet 4094 is connected to a personal computer IBM PC/XT clone through an RS-232 interface, so that the personal computer may work on the test data with various commercial software and with programs specially compiled for treatment of the SHPB test data. The theories relevant to the data treatment will be shown in next section.

Figure 2.2 is a schematic of the pressure bar arrangement for the Big Bar, with a Lagrange diagram above it illustrating the elastic wave propagation in the pressure

bars. Figure 2.3 shows an example of the axial strain signals versus time of a concrete test on the Big Bar, plotted by a Hewlett-Packard 7470A digital plotter from the stored signals in the Nicolet 4094. The incident pulse and the reflected pulse are measured by the gages on the incident bar, and the transmitted pulse is measured by the gage station on the transmitter bar. Also shown are records from two strain gages mounted on the specimen midway between its ends, one measuring axial surface strain  $\epsilon_x$  and one measuring transverse (hoop) strain  $\epsilon_\theta$ .

All the parameters of the 3-in-diameter SHPB system are tabulated in Table 2.1. The bar-wave velocity is a very important parameter of the system. It will be seen in the following section that the bar wave velocity is not only used to calculate the stresses according to the simple wave theory but also used in the wave-dispersion correction. In the dispersion correction, the phase velocity of every different frequency component of the pulse is a function of the frequency, the Poisson's ratio and the bar-wave velocity of the bar material. The bar-wave velocity was obtained by measuring the bar length and the time duration a stress pulse took for a round trip between the two ends of the bar.

From the bar-wave velocity and the time during which a compressive stress pulse travels from the gage station on the incident bar to the specimen-end face and then back to the gage station after reflection as a tensile stress pulse,

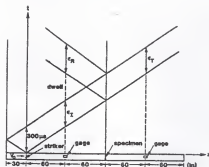


Figure 2.2 Schematic of the 3-in SHPB System and Its Lagrange Diagram

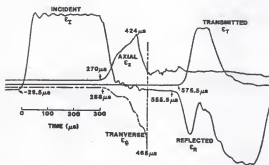
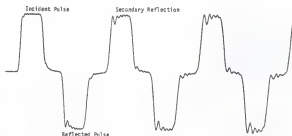
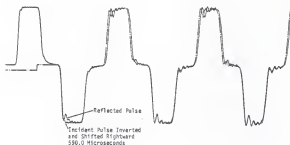


Figure 2.3 Strain Pulses in Pressure Bars and Axial and Transverse Specimen Surface Strains of the Test on Specimen WT3



(a) Records of the Incident Pulse and the Reflected Pulse in a Test Without a Specimen at the End of the Incident Bar



(b) The Incident Pulse and the Reflected Pulse Overlapping After the Reflected Pulse Is Inverted and Shifted 590  $\mu$ s

Figure 2.4 Measurement of the Time Duration for a Pulse to Take a Round Trip Between the Gage Station on the Incident Bar and the Specimen End Face

which was  $590.0 \mu\text{s}$ , the distance between the gage station and the specimen end was determined (see Figure 2.4).

The distance between the specimen end face and the gage station on the transmitter bar was measured in two ways. One was to get the incident bar and the transmitter bar in contact without a specimen between them and then to measure the time required a compressive pulse to go from the gage station on the incident bar to the gage station on the transmitter bar. The other was to measure the time required for a tensile pulse, which was caused by reflection of a compressive pulse at the dashpot end, to go from the gage station on the transmitter bar to the specimen-end face and then back to the gage station as a compressive pulse. The two tests acquired the same result, which was  $590.0 \mu\text{s}$ . The three tests proved that the two gage stations were equidistant to the specimen and that the wave travels between the specimen and either of the gage stations for  $295.0 \mu\text{s}$ . Figure 2.4 shows the time duration measurement on the incident bar, in which (a) is the original pulse record, and (b) presents the overlapping of the compressive pulse and the reflected (tensile) pulse after the operations of reversing and time shifting for  $590.0 \mu\text{s}$  on the Nicolet oscilloscope.



### 2.3 Stress Wave Theories Applied to the SHPB System

There are three simple cases in stress wave propagation: the one-dimensional linear strain case, the one-dimensional normal stress case and the pure shear stress case. The stress wave along a straight bar is the second case, where it is assumed that, in the Cartesian coordinates with the bar axis as one of its axes, no more components of the stress tensor at any material point than a normal component to the bar cross-section may exist, and that any bar cross-section perpendicular to the bar axis, which is the direction of the wave propagation, remains planar. It is sometimes accurate enough to analyze stress wave along a bar with the assumption of the one-dimensional stress wave if the diameter of the bar is small enough. The wave caused in plate impact is considered as the one-dimensional strain wave, if the thickness of the plate through which the wave penetrates is small enough in comparison with the length and width of the plate. The pure shear stress wave takes place in a tube or rod with an end being twisted. These are three extreme cases of the stress-wave propagation. Because of their simplicity, bar impact, plate impact and thin-walled tube twisting are commonly used to generate stress waves for determining material properties under dynamic loading. The compressive SHPB is an example of use of bar impact.

The one-dimensional stress wave theory is sometimes called the *simple stress-wave theory*, by which the strain rate of the specimen versus time and the stresses of the specimen at the end faces are easily obtained from the incident pulse, the reflected pulse and the transmitted pulse records in the SHPB test. For positive-direction elastic wave (away from the striker bar), with strain  $\epsilon$  considered positive in tension and particle velocity  $V$  positive in the direction from the impact end toward the specimen,

$$dV = -C_0 d\epsilon \quad (2.1)$$

and for the negative-direction wave,

$$dV = C_0 d\epsilon \quad (2.2)$$

where  $C_0$  is the elastic bar-wave speed. Therefore, if the strain pulses recorded at the gage stations are time shifted to the times when the pulses are at the specimen interfaces, one obtains with the zero initial conditions

$$V_I = -C_0(\epsilon_I - \epsilon_R) \quad (2.3)$$

$$V_T = C_0 \epsilon_T \quad (2.4)$$

where the subscript I denotes the incident-bar interface with the specimen and T denotes the transmitter-bar interface. The specimen strain rate is

$$\begin{aligned} \dot{\epsilon} &= \frac{V_T - V_I}{L_0} \\ &= -\frac{C_0}{L_0} (\epsilon_T - \epsilon_I + \epsilon_R) \end{aligned}$$

$$= \frac{C_0}{L_0} (\epsilon_I - \epsilon_R - \epsilon_T), \quad (2.5)$$

where  $L_0$  is the initial length of the specimen. The stresses  $\sigma_1$  and  $\sigma_2$  on the incident face and transmitter face of the specimen, if the specimen and the bar have the same diameter, are, respectively,

$$\sigma_1 = E(\epsilon_I + \epsilon_R) \text{ and} \quad (2.6)$$

$$\sigma_2 = E\epsilon_T. \quad (2.7)$$

With the existing programs and the microcomputer in the Nicolet 4094, the above manipulations can be easily completed. All the data of the concrete tests published (Malvern et al., 1985, 1986) are the results of these calculations, performed by the author of this dissertation. If the specimen is short enough, the stresses along the specimen would be approximately uniform after a very short time (a small fraction of the whole rise time of the loading), since a couple of reflections of the wave at the two end faces would be fulfilled. In that case,  $\sigma_1 = \sigma_2$  or  $\sigma_I + \sigma_R = \sigma_T$  after the "short time" so that it would be accurate enough to take  $E\epsilon_T$  as the stress measurement and  $-2C_0\epsilon_R/L_0$  as the strain-rate measurement. This approximation had been taken in the SHPB-test data treatment for a long time until advanced digital storage oscilloscopes appeared. It was also employed in the treatments of the mortar tests (Tang and Malvern, 1984).

In the event that the bar diameter is not very small, as in the Big Bar, stress dispersion due to the radial inertia may be serious. As a result, as shown in Figures 2.5 and 2.7,  $\sigma_1$  and  $\sigma_2$  obtained from the pulse records shown in Figure 2.3 according to Formulas 2.5 to 2.7 have apparent discrepancies.

The solution to the stress wave along an infinitely long cylinder with the stress-free lateral surface was given by Pochhammer in 1876 and by Chree in 1889, independently as an eigenvalue problem. The eigenvalue equation gives the phase velocities of sinusoidal longitudinal waves for any different frequency. Thus, a linearly-elastic bar of non-zero diameter is not a linear but a nonlinear system in delivering the longitudinal stress wave. In considering the SHPB system as a nonlinear system for delivering the longitudinal strain pulse with the strain pulse recorded at a strain-gage station as input of the system, the transfer function of the system and Fourier analysis of the input pulse have to be worked out in order to obtain the strain pulse away from the strain-gage station as the output from the system.

The correction of the stress-wave dispersion in the SHPB system based on Pochhammer-Chree analysis had been accomplished by Fourier series methods (Follansbee and Frantz, 1983; and Felice, 1985). At the University of Florida a program was developed (Malvern et al., 1987; Gong,

1988), using Fast Fourier Transform (FFT), which increased the efficiency of the calculation. All the wave-dispersion corrections done in this work were performed by the FFT method on the IBM PC/XT clone with a computation program in FORTRAN, which permits changing parameters of the SHPB system. The program gives dispersion-corrected stresses and velocities of the specimen at both the specimen end surfaces, and strain rate and strain of the specimen. After it was improved by the author of this dissertation, the program may be used either interactively or in a batch job.

Figures 2.5 to 2.8, which were formed with the data output from the program in 1-2-3<sup>1</sup>, edited in Freelance Plus, and then imported to WordPerfect 5.0 and printed out on a Hewlett-Packard LaserJet 500+ printer along with the text, compare the stresses  $\sigma_1$  and  $\sigma_2$  calculated without and with dispersion correction of the original test records shown in Figure 2.3. In Figures 2.5 and 2.6, the ordinates stress, strain rate and strain are in units of MPa, 1/10000 and 1/sec respectively, while the abscissa is time in units of microseconds, which gives time scale though the test does not start at the time origin set in the figures. In Figures 2.7 and 2.8, stress, strain rate and strain are in units of

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<sup>1</sup> 1-2-3, Freelance Plus and Wordperfect are software for the personal computer. 1-2-3 and Freelance Plus are registered trade marks of Lotus Development Corporation, and WordPerfect is a registered trade mark of WordPerfect Corporation.

MPa, 1/sec and 1/100 respectively. It is seen that  $\sigma_1$  and  $\sigma_2$  obtained after pulse correction coincide dramatically well.

The stress, strain rate and strain histories, and the stress-strain curve obtained with dispersion correction provide more reliable information, which, as will be seen later, not only confirms the strain-rate dependence of the compressive strength of concrete already reported (Malvern and Ross, 1986; Malvern et al., 1985, 1986), but also makes it possible to study the relation between stress and deformation before the maximum stress is reached.

It might be noteworthy to point out here that Pochhammer-Chree solution is not an exact but an approximate solution for finite cylinders, even if all the vibration frequencies are included in the solution. Only the strain history on the lateral surface of the bar at the gage location is measured in the practice of the SHPB test, but the strain-distribution histories over the cross section at the gage location, which should be necessary boundary conditions for a *well-posed wave propagation* problem, are not available. As a result, not only uniformity of the longitudinal displacements over the cross section at the gage station but also uniformity of the longitudinal displacements over the cross section at any other place have to be assumed. The planar cross-section assumption remains. The validity of the approximate dispersion correction

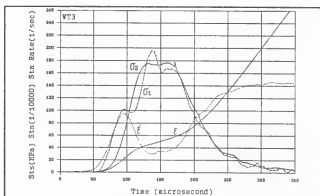


Figure 2.5 Stresses, Strain and Strain Rate Versus Time Curves of Specimen WT3 Calculated Without Dispersion Correction

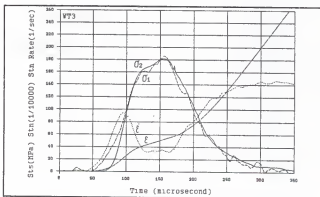


Figure 2.6 Stresses, Strain and Strain Rate Versus Time Curves of Specimen WT3 Calculated After Dispersion Correction

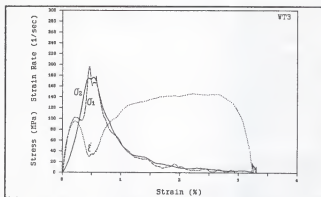


Figure 2.7 Stresses and Strain Rate Versus Strain Curves of Specimen WT3 Calculated Without Wave Dispersion Correction

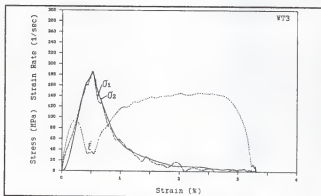


Figure 2.8 Stresses and Strain Rate Versus Strain Curves of Specimen WT3 Calculated After Wave Dispersion Correction



procedure is supported by the close agreement it gives between the calculated and measured pulses at two different stations on the same bar, as well as by the improved agreement between the two interface stresses already mentioned.

## 2.4 Dynamic Tests of Concrete and Mortar

### 2.4.1 Introduction

Unconfined specimens of mortar and of five kinds of high-strength concretes were tested in the SHPB systems. Mix designs of them are given in this section. The SHPB tests show that the maximum stress in the dynamic test increases with the strain rate at the maximum stress, and is higher than the static strength in each case. Detailed data of the tests on two concretes (WES and SRI limestone aggregate concretes) with wave dispersion correction, and analysis will be given in the following chapters.

### 2.4.2 Dynamic Tests of Mortar

A total of 127 specimens of four batches of 28-day cured mortar were tested on the Small Bar (Tang and Malvern, 1984). Figure 2.9 summarizes the results of the tests. The lower line shows the dependence of maximum stress for the 28-day cured specimens as

$$\max \sigma = f_c + A\dot{\epsilon} \quad (2.8)$$

where  $f_c$  is the static compressive strength;  $\dot{\epsilon}$  is the strain rate of the specimen at the maximum stress; and  $A = 6.56 \text{ psi/sec}$  ( $45.2 \times 10^{-3} \text{ MPa/sec}$ ). Each point marked with a solid circle is the average of 10 to 14 tests at the same impact speed. The static point on the curve is the average of 8 tests at strain rate of  $10^{-4}/\text{sec}$ . The point marked with an asterisk is for the larger standard ASTM specimens. The point marked  $\times$  is for a low speed where longitudinal cracks appeared but total crushing did not occur. The two upper lines are for a few specimens aged dry before testing. The extent of scatter in the 28-day test results is indicated in Figure 2.10 for two of the four batches. The drawbacks in inches marked in the figure are linear lengths pulled back by a pneumatic pump in the SHPB system to load the torsion spring in the test. The same drawback gives the same impact velocity.

The specimens tested were 0.4-in (10.2-mm) thick. They were cut from 28-day cured bars that had been prepared in the Civil Engineering Department, University of Florida with the assistance of Mr. Devo Seereeram, who was then an undergraduate student in that department. The mortar bar were cast in split PVC molds of nominal diameter 3/4 inch (19 mm) following a careful program of pouring and tamping to promote uniformity and reproducibility. After 24 hours, the bars were removed from the molds and placed in a wet

room in the Department of Civil Engineering, University of Florida to complete the 28-day cures. Standard ASTM 3 x 6 inch compression specimens were also cut from each batch. Table 2.2 shows mortar specifications. Each specimen was carefully measured and weighed and its ultrasonic wave speeds measured to check on reproducibility. Average density was 2070 kg/m<sup>3</sup>. From the ultrasonic and long-bar wave speeds the dynamic Poisson's ratio was 0.18.

Table 2.2 Mortar Specification

|           |                                                                                                                   |
|-----------|-------------------------------------------------------------------------------------------------------------------|
| Cement    | Portland Type I                                                                                                   |
| Aggregate | Sand Conforming to ASTM C33 grain size<br>$D_{max} = 2.36 \text{ mm}$ , $D_{90} = 0.33 \text{ mm}$ , $C_u = 2.11$ |
| Mix       | Tap water/sand/cement = 0.55/2.5/1.0<br>(by weight)                                                               |

#### 2.4.3. Dynamic Tests of Five Concretes

Five types of high-strength concrete were tested without confinement on the Big Bar. Part of the tests were performed with the assistance of Mr. Craig Hampson and Mr. Ronald Hunt, who were both undergraduate students in the Engineering Sciences Department, University of Florida. Some preliminary results have been published (Malvern and Ross, 1985, 1986; Malvern et al., 1985, 1986). All are specified as 14 ksi concrete, based on standard static unconfined compression tests. They differ mainly in the type of coarse aggregate used. Three of them were prepared,

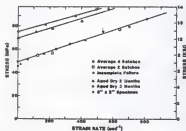


Figure 2.9 Maximum Stress Versus Strain Rate at Maximum Stress in Dynamic Tests of Mortar

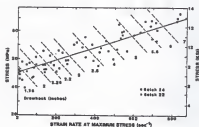


Figure 2.10 Maximum Stress Versus Strain Rate at Maximum Stress in Tests of Two Batches of Mortar

cured and cored from blocks by Terra Tek, Inc. of Salt Lake City. The three aggregates they used (maximum size 1/2 inch diameter) are designated as Andesite, Seattle gravel, and a lightweight aggregate called Solite. The fourth material with a manufactured limestone aggregate (maximum size 3/8 inch) was prepared at the U. S. Army Waterways Experiment Station (WES), cast in PVC pipe molds of three-inch diameter and cut to length after cures. The fifth concrete was prepared with limestone as the coarse aggregate by SRI International. All the specimens were further machined and ground at the University of Florida to ensure end face parallelism within 0.0005 inch. Table 2.3 and Table 2.4 show specifications of Terra Tek high-strength concretes and WES concrete respectively. Details of the mix of SRI concrete were not given.

The maximum value of average of  $\sigma_1$  and  $\sigma_2$ , which were calculated from test records the of 3-in-diameter and 3-in-long specimens without wave-dispersion correction, is plotted versus the average strain rate at the maximum average stress for the three Terra Tek concretes and WES concrete (Figures 2.11 and 2.12).

Figure 2.11 shows Andesite and Seattle gravel specimen results. The Andesite specimen (3-in diameter by 3-in long) had a static strength of 16.1 ksi and dynamic strengths varied from 20.4 up to 28.0 ksi at a strain rate of 77/sec. The inverted triangles in the lower group in Figure 2.11

represent specimens with Seattle gravel aggregate. The hollow inverted triangles represent tests in which the 120-in-long incident bar impacted the specimen directly in order to provide a longer pulse. Two of these direct impacts induced failure at strain rates as low as 3/sec and 4.8/sec. The dynamic strengths calculated by the conventional SHPB method varies from about 12 ksi at a strain rate of 10/sec to 18 ksi at 118/sec. In the tests with the incident bar directly impacting specimens, the gages on the incident bar recorded the pressure pulse induced by the direct impact. The particle velocity at the interface between the incident bar and the specimen is calculated as

$$V_I(t) = V_0 + C_0 R(t) \quad (2.9)$$

where  $V_0$  is the impact velocity,  $C_0$ , the bar-wave speed, and  $R(t)$ , the time-shifted strain gage record.

Figure 2.12 shows some preliminary tests of WES limestone aggregate specimen tests (solid circles) with dynamic strengths varying from 20.6 ksi at 9.31/sec to 29.9 ksi at 59.2/sec to compare the dynamic strengths of the WES concrete with those of the three kinds of concretes prepared at Terra Tek. Though all the data used in figures 2.11 and 2.12 were calculated with the simple stress-wave theory, without dispersion correction, they still give a good representation of how the dynamic strength of the concretes increases with the strain rate, since the dispersion correction does not change the calculated maximum stress and

Table 2.3 Mix for Terra Tek High Strength Concretes

| 0.75 cubic foot mix <sup>(a)</sup>          |               |
|---------------------------------------------|---------------|
| Water/Cement Ratio <sup>(b)</sup>           | 0.26          |
| Water                                       | 7.52 lb @ 9°C |
| DARACAM 100 Superplasticizer <sup>(c)</sup> | 425 cc        |
| Type II Portland Cement                     | 18.33 lb      |
| Hana Microsilicate (Silica Fume)            | 3.9 lb        |
| Flyash                                      | 3.9 lb        |
| Fine Aggregate (#4 Concrete Sand)           | 39.9 lb       |
| Coarse Aggregate <sup>(d)</sup>             | 41.67 lb      |

<sup>(a)</sup> Slump varies from 4 to 9 inches.

<sup>(b)</sup> Varies depending on water/cement ratio and absorption properties of aggregates from 0.24 to 0.27.

<sup>(c)</sup> Varies from 400 to 550 cc.

<sup>(d)</sup> Andesite or rounded river gravel

Table 2.4 Mix for Waterways Experiment Station Concrete

| 1 cubic yard mix (Slump 8.5 inches)                                         |         |
|-----------------------------------------------------------------------------|---------|
| Water/Cement Ratio (based on total cementitious material)                   | 0.27    |
| Type I Portland Cement                                                      | 850 lb  |
| Silica Fume                                                                 | 150 lb  |
| Fine Aggregate (Manufactured limestone from Vulcan Materials, Calera, Ala.) | 1860 lb |
| Coarse Aggregate (Manufactured limestone, max size 3/8 inch)                | 1008 lb |
| Water                                                                       | 270 lb  |
| High Range Water Reducing Add Mixture                                       | 20 lb   |
| DAXAD-19-2% by weight of cementitious material (superplasticizer)           |         |

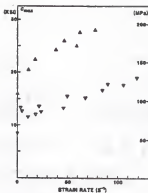


Figure 2.11 Maximum Stress Versus Strain Rate at Maximum Stress: Andesite (upper) and Seattle Gravel Concrete (lower inverted triangles)

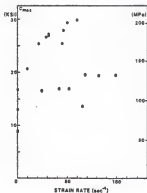


Figure 2.12 Maximum Stress Versus Strain Rate at Maximum Stress: WES Limestone Aggregate (circles) and Solite Lightweight Aggregate (squares)



strain rate at the maximum stress much. Many tests on the WES concrete were performed, and all the pulse records for WES and SRI concrete tests have been dispersion corrected. The details of unconfined WES and SRI tests and discussion will be presented in next two chapters.

CHAPTER 3  
DYNAMIC TESTS AND STRAIN-RATE DEPENDENCE OF THE  
COMPRESSIVE STRENGTH AND YIELD STRESS OF CONCRETE

3.1 Dynamic Tests of WES and SRI Concretes

The results of the 35 WES tests and 32 SRI tests, performed with the 3-inch SHPB system, were all calculated from the dispersion corrected pulses and are summarized in Tables 3.1 to 3.4. In the tables and figures in this chapter, the compressive stress, strain rate and strain are taken as positive. For all the specimens, the stresses at the incident and transmitter interfaces,  $\sigma_1$  and  $\sigma_2$ , have coincided since well before they reach the maximum values. As a uniform standard, the maximum  $\sigma_2$  value and the strain rate at the maximum  $\sigma_2$  of every test are picked and listed in these tables to show the increase of the dynamic compressive strength with the strain rate. A higher firing pressure (pressure in the firing chamber) may sometimes produce a lower impact speed of the striker bar against the incident bar. Impactor speeds were determined from the amplitude of the recorded incident strain pulse.

WES specimens have identifications with letter W as the initial or the last letter, e.g. W24 and 02W, and SRI

specimens have identifications with letter S as the initial, e.g. S64. The critical strain is the strain at the maximum stress (or at the beginning of a maximum-stress plateau, e.g. in Figure 3.1). The symbols in the last column are for comments, as will be explained later in the text.

Table 3.1 lists tests of 27 WES specimens of nominal length 3 inches. These specimens had been roughly 3-inch long, as provided by the U. S. Army Waterways Experiment Station, and were further ground at the University of Florida to ensure that the end surfaces were parallel within 0.0005 inch, except for specimens with a letter T following W in their identifications, e.g. WT3, which had had sufficient end-face parallelism already, as provided by WES, with length of 3.000 inches (0.0672 m). The final lengths of the specimens further ground at the University of Florida ranged from 2.638 inches (0.0670 m) to 2.868 inches (0.0728 m). The specimens whose test results are shown in Table 3.2 were nominally 1.5-inch long. They were cut off from the roughly 3-inch long specimens and then ground. Their actual lengths ranged from 1.261 inches (0.03203 m) to 1.392 inches (0.03536 m). Table 3.3 tabulates the details of 27 tests of 1.5-inch long (0.0386 m) SRI specimens, and Table 2.9, the details of 4 tests of 3-inch (0.0762 m) SRI specimens and 1 test of a 2.86 inch-long (0.0726 m) specimen. All the SRI specimens were prepared and machined at the SRI and had well-parallel end faces except that the 2.86-inch long

Table 3.1 Details of 27 Tests on 3-inch-long WES Specimens

| I.D. | Firing Pressure (psi) | Impact Speed (in/s) | Maximum Stress (ksi) (MPa) |     | Strain Rate @ Max Stress (1/sec) | Critical Strain (%) |    |
|------|-----------------------|---------------------|----------------------------|-----|----------------------------------|---------------------|----|
| W24  | 180                   | 276                 | 19.1                       | 132 | 5.2                              | 0.36                | LC |
| W03  | 200                   | 259                 | 19.3                       | 133 | 0.0                              | 0.32                | NF |
| W02  | 210                   | 333                 | 22.9                       | 158 | 8.1                              | 0.40                | LC |
| W42  | 225                   | 324                 | 21.9                       | 151 | 11.6                             | 0.41                |    |
| W10  | 250                   | 307                 | 20.9                       | 144 | 9.5                              | 0.41                |    |
| W43  | 250                   | 377                 | 23.6                       | 163 | 18.9                             | 0.43                |    |
| W20  | 250                   | 380                 | 24.7                       | 170 | 15.8                             | 0.44                | LC |
| W21  | 275                   | 390                 | 25.4                       | 175 | 15.2                             | 0.44                |    |
| W32  | 275                   | 428                 | 24.5                       | 169 | 37.7                             | 0.50                |    |
| W36  | 275                   | 442                 | 25.5                       | 176 | 32.7                             | 0.53                |    |
| W08  | 300                   | 400                 | 25.7                       | 177 | 21.2                             | 0.44                |    |
| W40  | 300                   | 434                 | 26.0                       | 179 | 33.4                             | 0.49                | LC |
| WT1  | 350                   | 439                 | 26.4                       | 182 | 30.0                             | 0.53                |    |
| W09  | 350                   | 450                 | 25.2                       | 174 | 41.2                             | 0.52                |    |
| WT3  | 350                   | 451                 | 26.4                       | 182 | 34.4                             | 0.52                | LC |
| W26  | 350                   | 468                 | 25.5                       | 176 | 41.7                             | 0.49                |    |
| W11  | 400                   | 492                 | 25.2                       | 174 | 43.32                            | 0.48                |    |
| WT4  | 400                   | 505                 | 25.1                       | 173 | 45.3                             | 0.52                |    |
| W44  | 400                   | 530                 | 27.0                       | 186 | 53.0                             | 0.50                | LC |
| W15  | 450                   | 544                 | 28.9                       | 200 | 49.6                             | 0.47                |    |
| W46  | 450                   | 588                 | 29.1                       | 201 | 69.2                             | 0.54                | LC |
| W19  | 500                   | 582                 | 29.6                       | 204 | 63.2                             | 0.47                |    |
| W39  | 500                   | 621                 | 29.4                       | 203 | 92.9                             | 0.56                | LC |
| W37  | 550                   | 625                 | 29.0                       | 200 | 105                              | 0.54                |    |
| W30  | 600                   | 665                 | 29.9                       | 206 | 125                              | 0.57                |    |
| W35  | 600                   | 683                 | 31.9                       | 220 | 125                              | 0.57                |    |
| W28  | 600                   | 690                 | 30.3                       | 209 | 135                              | 0.61                | LC |

Table 3.2 Details of 8 Tests on 1.5-inch-long WES Specimens

| I.D. | Firing Pressure (psi) | Impact Speed (in/s) | Maximum Stress (ksi) (MPa) |     | Strain Rate @ Max Stress (1/sec) | Critical Strain (%) |    |
|------|-----------------------|---------------------|----------------------------|-----|----------------------------------|---------------------|----|
| 03W  | 200                   | 231                 | 17.1                       | 118 | 0.0                              | 0.52                | NF |
| 04W  | 300                   | 376                 | 23.2                       | 160 | 42.6                             | 0.45                |    |
| 05W  | 350                   | 469                 | 27.8                       | 192 | 90.2                             | 0.61                |    |
| 01W  | 400                   | 506                 | 29.1                       | 201 | 107                              | 0.58                |    |
| 06W  | 400                   | 519                 | 29.3                       | 202 | 132                              | 0.68                |    |
| 07W  | 450                   | 569                 | 29.1                       | 201 | 184                              | 0.72                |    |
| 08W  | 500                   | 605                 | 31.1                       | 216 | 186                              | 0.70                |    |
| 02W  | 600                   | 674                 | 33.5                       | 231 | 219                              | 0.68                |    |

Table 3.3 Details of 27 Tests on 1.5-inch-long SRI Specimens

| I.D. | Firing Pressure (psi) | Impact Speed (in/s) | Maximum Stress (ksi) (MPa) |     | Strain Rate @ Max Stress (1/sec) | Critical Strain (%) |    |
|------|-----------------------|---------------------|----------------------------|-----|----------------------------------|---------------------|----|
| S41  | 150                   | 215                 | 15.7                       | 108 | 0.0                              | 0.34                | NF |
| S31  | 175                   | 262                 | 19.0                       | 131 | 0.0                              | 0.19                | NF |
| S23  | 185                   | 274                 | 19.4                       | 134 | 4.7                              | 0.39                |    |
| S51  | 185                   | 283                 | 19.9                       | 137 | 5.8                              | 0.47                |    |
| S12  | 200                   | 294                 | 21.2                       | 146 | 9.6                              | 0.46                |    |
| S32  | 200                   | 299                 | 20.6                       | 142 | 6.9                              | 0.49                | L  |
| S15  | 200                   | 316                 | 20.9                       | 144 | 20.6                             | 0.64                | CC |
| S33  | 225                   | 333                 | 22.0                       | 152 | 25.3                             | 0.49                |    |
| S22  | 250                   | 368                 | 22.8                       | 157 | 41.4                             | 0.58                |    |
| S63  | 250                   | 369                 | 22.6                       | 156 | 47.8                             | 0.60                |    |
| S25  | 250                   | 376                 | 22.3                       | 154 | 53.3                             | 0.71                | CC |
| S11  | 300                   | 418                 | 25.1                       | 173 | 42.0                             | 0.63                | C  |
| S45  | 300                   | 441                 | 24.4                       | 168 | 72.1                             | 0.60                | CC |
| S54  | 300                   | 446                 | 26.1                       | 180 | 48.2                             | 0.65                |    |
| S55  | 350                   | 464                 | 28.9                       | 199 | 62.9                             | 0.50                |    |
| S43  | 350                   | 469                 | 26.0                       | 179 | 73.5                             | 0.66                |    |
| S21  | 400                   | 500                 | 26.1                       | 180 | 116                              | 0.69                | L  |
| S14  | 400                   | 535                 | 28.3                       | 195 | 121                              | 0.70                | C  |
| S13  | 400                   | 538                 | 26.2                       | 181 | 137                              | 0.94                | CC |
| S61  | 450                   | 546                 | 28.1                       | 194 | 146                              | 0.71                |    |
| S52  | 450                   | 584                 | 28.6                       | 197 | 157                              | 0.73                | C  |
| S62  | 500                   | 578                 | 31.3                       | 216 | 145                              | 0.64                |    |
| S44  | 500                   | 603                 | 29.6                       | 204 | 164                              | 1.01                | CC |
| S34  | 600                   | 632                 | 31.4                       | 217 | 180                              | 0.67                |    |
| S24  | 600                   | 681                 | 30.9                       | 213 | 226                              | 0.85                | CC |
| S64  | 600                   | 684                 | 30.5                       | 210 | 215                              | 0.95                | CC |
| S53  | 600                   | 690                 | 30.9                       | 213 | 214                              | 1.03                | CC |

Table 3.4 Details of 5 Tests on 3-inch-long SRI Specimens

| I.D. | Firing Pressure (psi) | Impact Speed (in/s) | Maximum Stress (ksi) (MPa) |     | Strain Rate @ Max Stress (1/sec) | Critical Strain (%) |    |
|------|-----------------------|---------------------|----------------------------|-----|----------------------------------|---------------------|----|
| SB * | 200                   | 317                 | 21.0                       | 145 | 10.4                             | 0.49                | CC |
| SE   | 300                   | 456                 | 25.8                       | 178 | 39.1                             | 0.56                |    |
| SD   | 400                   | 539                 | 25.5                       | 176 | 56.2                             | 0.55                | CC |
| SF   | 500                   | 610                 | 27.0                       | 186 | 86.9                             | 0.55                |    |
| SA   | 600                   | 697                 | 29.4                       | 203 | 114                              | 0.60                | CC |

\* 2.860 inches (0.0726 m) long

specimen was further ground at the University of Florida to ensure the end face parallelism within 0.0005 inch.

Strain gages were mounted on some specimens midway between the ends and used to measure transverse (hoop) strain and axial surface strain during the SHPB test. The specimens tested with directly-mounted strain gages are marked in the tables. The last column in each table is for remarks, among which letters NF mean the specimen did not fracture, and one letter L indicates one strain gage mounted on the specimen longitudinally and one letter C indicates one strain gage mounted circumferentially. Except non-fractured specimens and Specimen W24, which fractured partly, all the specimens were destroyed into small pieces in the SHPB tests.

The stresses, strain rate and strain versus time curves of Specimen WT3 and its stresses and strain rate versus strain curves have been shown in Figures 2.5 and 2.7 respectively in the previous chapter. Figures 3.1 to 3.28 give more samples of those curves of WES and SRI specimens, which were tested with diverse firing-chamber pressures. A specimen identification appears in an upper corner of every figure. All the figures for the stresses, strain rate and strain versus time curves are gridded, with stresses in units of MPa, strain rate in 1/sec, strain in 1/10000 and time in microsecond, while the figures for the stresses and strain rate versus strain curves are not gridded, with stress in units of MPa, strain rate in 1/sec also but strain

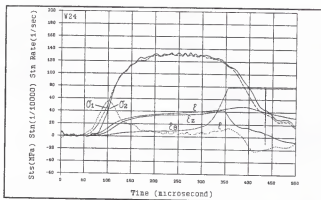


Figure 3.1 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W24 Calculated After Wave Dispersion Correction

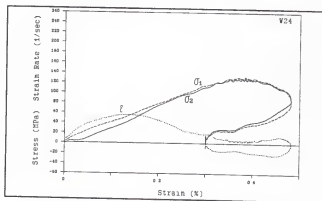


Figure 3.2 Stresses and Strain Rate Versus Strain Curves of Specimen W24 Calculated After Wave Dispersion Correction

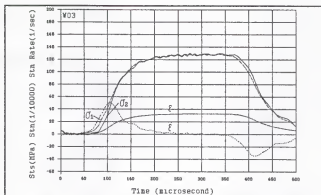


Figure 3.3 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W03 Calculated After Wave Dispersion Correction

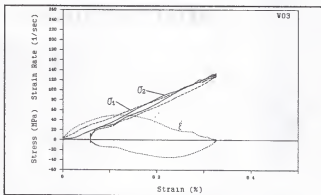


Figure 3.4 Stresses and Strain Rate Versus Strain Curves of Specimen W03 Calculated After Wave Dispersion Correction



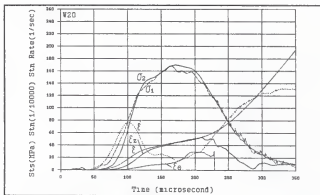


Figure 3.5 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W20 Calculated After Wave Dispersion Correction

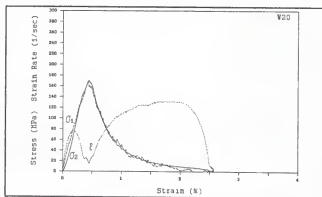


Figure 3.6 Stresses and Strain Rate Versus Strain Curves of Specimen W20 Calculated After Wave Dispersion Correction

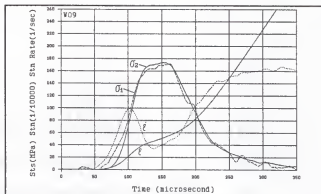


Figure 3.7 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W09 Calculated After Wave Dispersion Correction

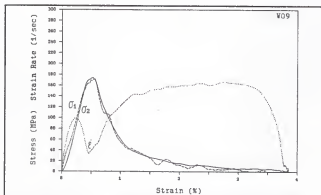


Figure 3.8 Stresses and Strain Rate Versus Strain Curves of Specimen W09 Calculated After Wave Dispersion Correction

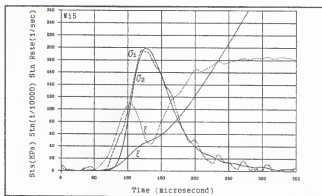


Figure 3.9 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W15 Calculated After Wave Dispersion Correction

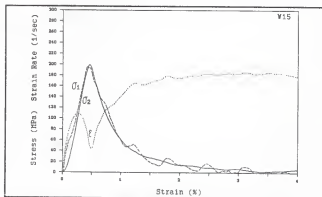


Figure 3.10 Stresses and Strain Rate Versus Strain Curves of Specimen W15 Calculated After Wave Dispersion Correction

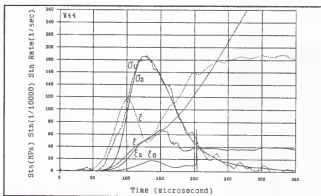


Figure 3.11 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W44 Calculated After Wave Dispersion Correction

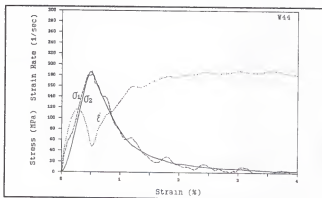


Figure 3.12 Stresses and Strain Rate Versus Strain Curves of Specimen W44 Calculated After Wave Dispersion Correction

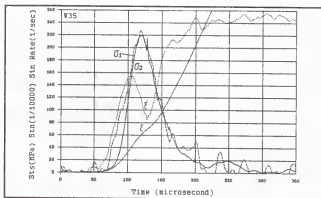


Figure 3.13 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W35 Calculated After Wave Dispersion Correction

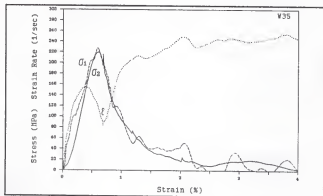


Figure 3.14 Stresses and Strain Rate Versus Strain Curves of Specimen W35 Calculated After Wave Dispersion Correction

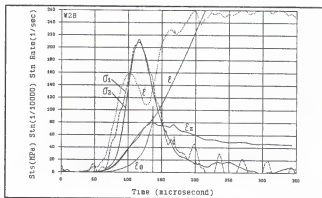


Figure 3.15 Stresses, Strain Rate and Strain Versus Time Curves of Specimen W28 Calculated After Wave Dispersion Correction

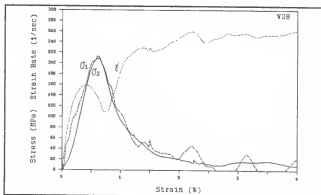


Figure 3.16 Stresses and Strain Rate Versus Strain Curves of Specimen W28 Calculated After Wave Dispersion Correction

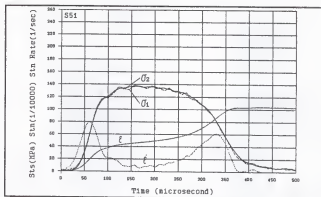


Figure 3.17 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S51 Calculated After Wave Dispersion Correction

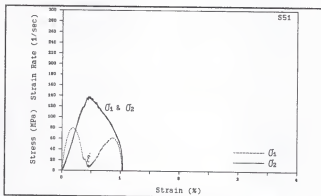


Figure 3.18 Stresses and Strain Rate Versus Strain Curves of Specimen S51 Calculated After Wave Dispersion Correction

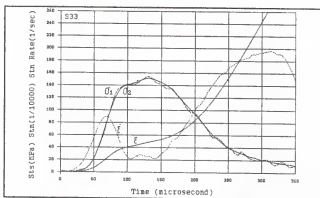


Figure 3.19 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S33 Calculated After Wave Dispersion Correction

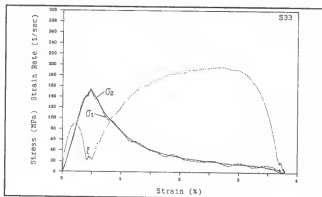


Figure 3.20 Stresses and Strain Rate Versus Strain Curves of Specimen W24 Calculated After Wave Dispersion Correction



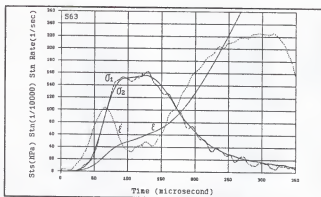


Figure 3.21 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S63 Calculated After Wave Dispersion Correction

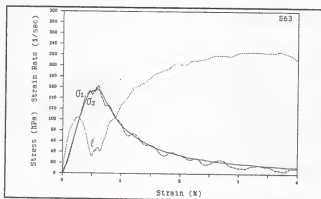


Figure 3.22 Stresses and Strain Rate Versus Strain Curves of Specimen S63 Calculated After Wave Dispersion Correction

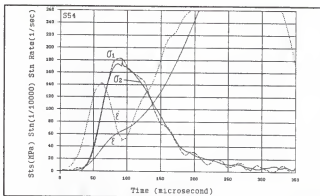


Figure 3.23 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S54 Calculated After Wave Dispersion Correction

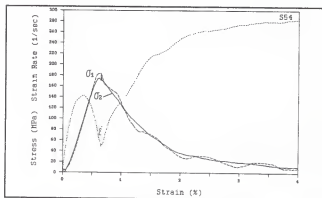


Figure 3.24 Stresses and Strain Rate Versus Strain Curves of Specimen S54 Calculated After Wave Dispersion Correction

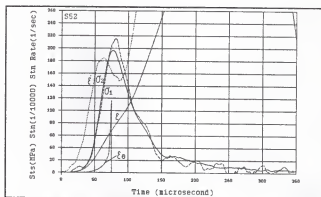


Figure 3.25 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S52 Calculated After Wave Dispersion Correction

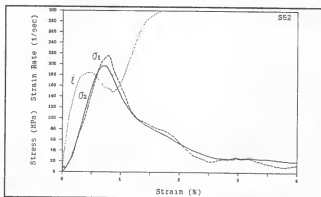


Figure 3.26 Stresses and Strain Rate Versus Strain Curves of Specimen S52 Calculated After Wave Dispersion Correction

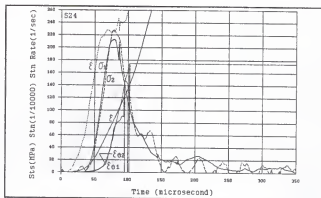


Figure 3.27 Stresses, Strain Rate and Strain Versus Time Curves of Specimen S24 Calculated After Wave Dispersion Correction

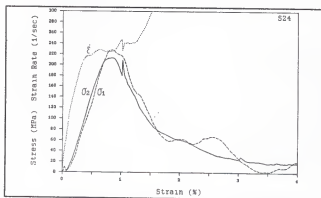


Figure 3.28 Stresses and Strain Rate Versus Strain Curves of Specimen S24 Calculated After Wave Dispersion Correction

in 1/100. The coordinates of the figures of the same kind, gridded or ungridded, are in the same scales so that the curves for different specimens can be conveniently compared, except for the figures for the specimens W24 and W03 tested at very low impact speeds, where the maximum strains are much smaller and negative values of the strain rate appear upon unloading. The unloading is caused by the end of the SHPB loading pulse and not by general failure of the specimen in the case of W03, which was not fractured, and W24, which was only partly fractured. Further analysis of these tests is given in the following sections.

### 3.2 Strain-Rate Dependence of the Strength and Yield Stress of Concrete

#### 3.2.1 Strain-Rate Dependence of the Strength

Strain-Rate Dependence of the strength of mortar and concrete has been shown in Subsection 2.2.3, but the data were calculated without the wave-dispersion correction. The maximum stress  $\sigma_2$  and the strain rate  $\dot{\epsilon}$  at the maximum stress  $\sigma_2$  of every test for WES and SRI specimens, calculated after the SHPB records were wave-dispersion corrected, were tabulated in Tables 3.1 to 3.4. In low-speed impact tests, e.g. W24 (Figures 3.1 and 3.2), stress remained approximately constant for a period of time after maximum stress was reached to form a maximum-stress plateau in the stress versus time curve. This phenomenon is defined

as delayed fracture. The strain value at the beginning of the stress plateau was recorded as the critical strain listed in Tables 3.1 and 3.4 if the test presented delayed fracture, because the apparent Poisson's ratio increased rapidly at the beginning of the stress plateau, as will be shown later. These data are summarized in Figures 3.29 to 3.34 to show the increase of strength and critical strain of concrete with the strain rate. No considerable size effect is observed. For either WES concrete or SRI concrete, there are two figures for strain-rate dependence of the strength, in one of which the stress scale is in units of MPa and in the other, ksi. Either a semilogarithmic regression or power-law regression fits for WES and SRI concretes in the following expressions.

$$\text{Max } \sigma_2 = A + B \ln (\dot{\epsilon}/\dot{\epsilon}_0) \quad (3.1)$$

$$\text{Max } \sigma_2 = a (\dot{\epsilon}/\dot{\epsilon}_0)^b \quad (3.2)$$

For  $\dot{\epsilon}_0 = 1/\text{sec}$  (compression as positive) the coefficients are tabulated in Table 3.5. These coefficients were obtained by the least square method using only data from the 33 WES specimens and the 30 SRI specimens that were visibly fractured, although the points of the specimens that appeared intact after being tested are also close to the regression curves.

There are two questions about the regression. One is the reasonability of the choice of the strain rate at the maximum stress as the abscissa, and the other is whether it

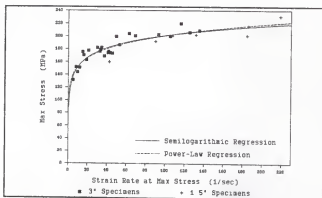


Figure 3.29 Maximum Stress  $\sigma_2$  Versus Strain Rate  $\dot{\epsilon}$  at Maximum  $\sigma_2$  with Maximum  $\sigma_2$  in Units of MPa for WES Specimens

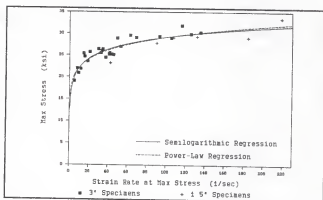


Figure 3.30 Maximum Stress  $\sigma_2$  Versus Strain Rate  $\dot{\epsilon}$  at Maximum  $\sigma_2$  with Maximum  $\sigma_2$  in Units of ksi for WES Specimens

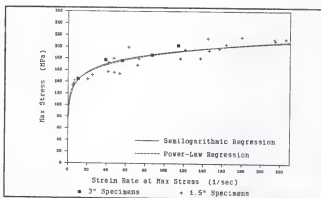


Figure 3.31 Maximum Stress  $\sigma_2$  Versus Strain Rate  $\dot{\epsilon}$  at Maximum  $\sigma_2$  with Maximum  $\sigma_2$  in Units of MPa for SRI Specimens

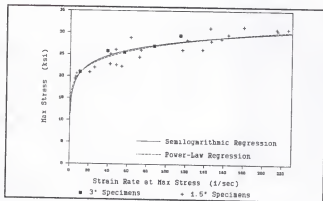


Figure 3.32 Maximum Stress  $\sigma_2$  Versus Strain Rate  $\dot{\epsilon}$  at Maximum  $\sigma_2$  with Maximum  $\sigma_2$  in Units of ksi for SRI Specimens



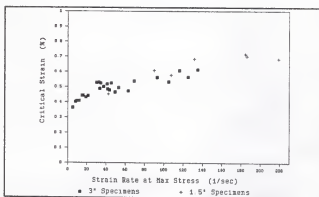


Figure 3.33 Critical Strain Versus Strain Rate at Maximum Stress for WES Specimens

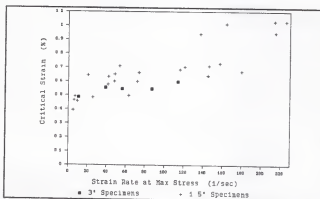


Figure 3.34 Critical Strain Versus Strain Rate at Maximum Stress for SRI Specimens

Table 3.5 Regression Coefficients for Strain-Rate Dependence of Compressive Strength of WES and SRI Concretes

| Function Type   | Coeff. | WES                    | SRI                    |
|-----------------|--------|------------------------|------------------------|
| Semilogarithmic | A      | 103 MPa<br>(14.9 ksi)  | 93.6 MPa<br>(13.6 ksi) |
|                 | B      | 21.3 MPa<br>(3.09 ksi) | 21.1 MPa<br>(3.03 ksi) |
| Power Law       | a      | 116 MPa<br>(16.8 ksi)  | 108 MPa<br>(15.6 ksi)  |
|                 | b      | 0.119                  | 0.122                  |

is accidental that the maximum stresses that appeared in the non-fracture tests are so close to the regression curves that are based on the fracture tests. More analysis shows (1) that the fracture is an accumulative process and (2) that the specimen stress after yield but in loading is determined by the inelastic strain rate according to (3.1) or (3.2) if  $\max \sigma_2$  is replaced by general  $\sigma$ . These results give the first question a positive answer and the second question a negative answer. Details of the analysis will be seen in Chapter 4.

Biaxial and triaxial static tests have indicated that the hydrostatic pressure increases the compressive strength (See, e.g., Balmer, 1949; Kupfer, 1969, 1973). In the uniaxial SHPB tests of unconfined specimens, the hydrostatic pressure (calculated as one third of the magnitude of the maximum longitudinal normal stress) reached the values

higher than one third of the static compressive strength. But for any given level of stress below the static strength, the hydrostatic component was the same as at the same level of stress in a static test, so that the higher dynamic strength cannot be attributed to a higher hydrostatic component if the stress is uniaxial.

Radial inertia effects in geotechnical materials, which may be mistaken for strain-rate effects, were discussed by Glenn and Janach (1977) and by Young and Powell (1979). In their tests it appears that failure occurred during the first passage of the stress wave through the specimen. In the SHPB tests reported here failure occurs only after many wave reflections back and forth between the specimen interfaces. The analysis of the strains measured by the strain gages mounted directly on the specimens will show in Section 3.3 that the radial inertia in the SHPB tests contributes very small radial pressure, which would not bring about significant increase of the compressive strength of the concrete.

### 3.2.2 Strain-Rate Dependence of the Yield Stress

All the stress-strain curves shown in the previous section have a linear part from the start of loading. For the tests where the specimens remained intact or were only partly fractured, e.g. W03 and W24 (Figures 3.2 and 3.4), during unloading the stress went down with strain along a

linear path, which was approximately parallel to the linear loading path. The specimens with no fracture observed after the test resumed their original sizes. These phenomena indicate that the strain may be decomposed into two parts, linear elastic strain and inelastic strain, to describe behavior of WES and SRI concretes. The strain records from the strain gages directly mounted on the specimen helped determine the elastic limit or yield point. Figures 3.35 to 3.38 show the lateral (hoop) strain versus longitudinal strain relations of four WES specimens. The lateral strain and longitudinal strain are both from the strain gages mounted on the specimen, designated as  $\epsilon_\theta$  and  $\epsilon_z$ , respectively. For SRI specimens, strain gages were mainly mounted in the lateral direction, since the tests on WES specimens had proved that  $\epsilon_z$  coincided well with  $\epsilon_r$ , which was obtained with the SHPB system. The lateral strain  $\epsilon_\theta$  versus strain  $\epsilon_r$  relations of two SRI specimen are shown in Figures 3.39 and 3.40. In Figure 3.40,  $\epsilon_{\theta 1}$  and  $\epsilon_{\theta 2}$  are respectively records from two strain gages mounted on the specimen both in the lateral direction but apart  $180^\circ$  from each other. The slope of the  $\epsilon_\theta$  versus  $\epsilon_z$  or  $\epsilon_r$  curve keeps constant until a turning point marked with an arrow (the lower one of the two arrows in each figure). This point is defined as the yield point, below which the material is of a constant Young's modulus and a constant Poisson's ratio. Tables 3.6 and 3.7 list yield stresses, which were picked

from the  $\sigma_2$  record at the time of the yield point, and the average strain rate  $\Delta\epsilon/\Delta t$  from the time  $\sigma_2$  starts to the yield point of nine WES and eight SRI specimens. In specimens that were tested with the same firing pressure, e.g. S24, S64 and S53 all tested with firing pressure of 600 psi, very similar results were obtained, so only one specimen was listed in the table. In some SRI specimens other than the eight specimens listed, the mounted gages started recording later than the SHPB system so data from these mounted gages were discarded. Yield stresses for W02 and WT3 were not obtained because the  $\epsilon_1$  record dropped down after a straight-line segment. As shown in Tables 3.6 and 3.7, the dynamic yield stress values obtained even exceed the static compressive strength. Figure 3.41 plots these data and shows that yield stress increases with the average strain rate. A linear regression was made for WES specimens and SRI 1.5-inch specimens respectively with the yield stress  $\sigma_y$ , expressed in terms of the average strain rate as

$$\sigma_y = C + D \dot{\epsilon}_{ave} \quad (3.3)$$

where constants C and D are listed in Table 3.8.

At the yield point the slope of the  $\epsilon_1$  versus  $\epsilon_2$  or  $\epsilon$  curve jumped and beyond the yield point it was again approximately constant at a greater value than the initial Poisson's ratio until another turning point, marked by the upper arrow in the figures. Just at the time of the point marked by the upper arrow, the maximum stress  $\sigma_2$  was

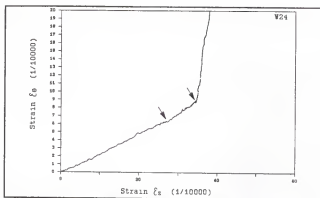


Figure 3.35 Lateral Strain  $\epsilon_y$  Versus Longitudinal Strain  $\epsilon_x$  Relation of Specimen W24

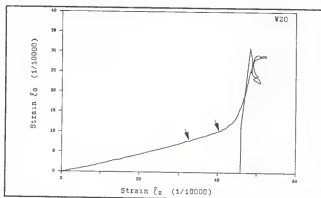


Figure 3.36 Lateral Strain  $\epsilon_y$  Versus Longitudinal Strain  $\epsilon_x$  Relation of Specimen W20

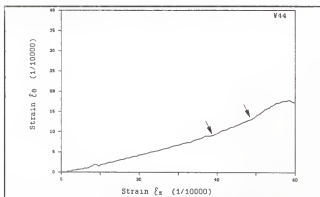


Figure 3.37 Lateral Strain  $\epsilon_y$  Versus Longitudinal Strain  $\epsilon_x$  Relation of Specimen W44

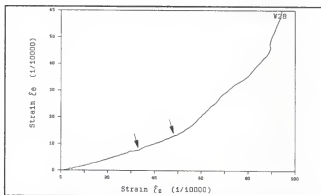


Figure 3.38 Lateral Strain  $\epsilon_y$  Versus Longitudinal Strain  $\epsilon_x$  Relation of Specimen W28

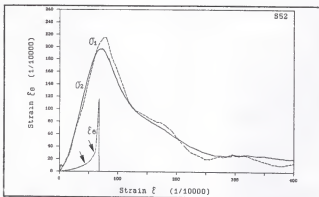


Figure 3.39 Lateral Strain  $\epsilon_0$  Versus Longitudinal Strain  $\epsilon$  Relation of Specimen S52

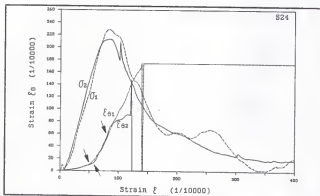


Figure 3.40 Lateral Strain  $\epsilon_0$  Versus Longitudinal Strain  $\epsilon$  Relation of Specimen S24



Table 3.8<sup>6</sup> Dynamic Compressive Yield Stress,  
Young's Modulus and Poisson's Ratio of WES Specimens

| I.D. | Yield Stress<br>(MPa) (ksi) |      | Young's Modulus<br>(GPa) (ksi) |      | Poisson's<br>Ratio | Ave Strn Rate<br>(1/sec) |
|------|-----------------------------|------|--------------------------------|------|--------------------|--------------------------|
| W24  | 114                         | 16.5 | 38.7                           | 5620 | 0.237              | 31.8                     |
| W02  | -                           | -    | 39.6                           | 5740 | 0.256              | 39.9                     |
| W20  | 148                         | 21.5 | 42.9                           | 6210 | 0.240              | 53.9                     |
| W40  | 146                         | 21.2 | 43.3                           | 6810 | 0.214              | 74.5                     |
| WT3  | -                           | -    | 40.6                           | 5890 | 0.306              | 60.4                     |
| W44  | 152                         | 22.0 | 42.3                           | 6140 | 0.231              | 88.6                     |
| W46  | 156                         | 22.6 | 41.2                           | 5970 | 0.286              | 72.0                     |
| W39  | 161                         | 23.4 | 40.1                           | 5820 | 0.307              | 81.8                     |
| W28  | 158                         | 22.9 | 39.0                           | 5650 | 0.291              | 88.3                     |

Table 3.9<sup>7</sup> Dynamic Compressive Yield Stress,  
Young's Modulus and Poisson's Ratio of SRI Specimens

| I.D. | Yield Stress<br>(MPa) (ksi) |      | Young's Modulus<br>(GPa) (ksi) |      | Poisson's<br>Ratio | Ave Strn Rate<br>(1/sec) |
|------|-----------------------------|------|--------------------------------|------|--------------------|--------------------------|
| S15  | 131                         | 19.0 | 26.1                           | 3780 | 0.274              | 78.1                     |
| S25  | 137                         | 19.9 | 26.0                           | 3770 | 0.261              | 79.4                     |
| S45  | 131                         | 19.0 | 32.9                           | 4770 | 0.249              | 107                      |
| S14  | 159                         | 23.1 | 33.8                           | 4900 | 0.303              | 137                      |
| S52  | 153                         | 22.2 | 31.2                           | 4520 | 0.293              | 114                      |
| S24  | 172                         | 24.9 | 30.0                           | 4330 | 0.254              | 136                      |
| SB   | 119                         | 17.3 | 31.0                           | 4490 | 0.276              | 44.4                     |
| SA   | 142                         | 20.6 | 34.6                           | 5020 | 0.248              | 63.4                     |

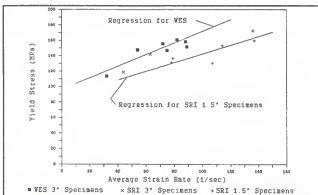


Figure 3.41 Yield Stress Versus Average Strain Rate for WES and SRI Specimens

Table 3.8 Regression Coefficients for Strain-Rate Dependence of Compressive Yield Stress of WES and SRI Concretes

| Coeff. | WES                              | SRI                               |
|--------|----------------------------------|-----------------------------------|
| C      | 96.4 MPa<br>(14.0 ksi)           | 86.0 MPa<br>(12.5 ksi)            |
| D      | 0.753 MPa·sec<br>(0.109 ksi·sec) | 0.563 MPa·sec<br>(0.0817 ksi·sec) |

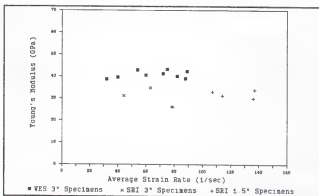


Figure 3.42 Young's Modulus Versus Average Strain Rate for WES and SRI Specimens

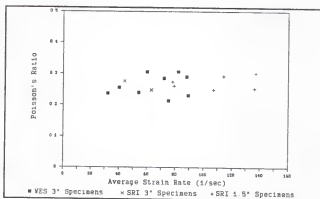


Figure 3.43 Poisson's Ratio Versus Average Strain Rate for WES and SRI Specimens

reached. In the delayed-fracture test, e.g. W24, this turning point corresponded to the beginning of the maximum-stress plateau. From the maximum stress on, the lateral strain  $\epsilon_2$  increases greatly, and cracks in the specimen were believed to be developing and connecting with one another rapidly, as the specimen approached complete failure.

Values of Young's modulus and Poisson's ratio obtained below the yield point are also listed in Tables 3.6 and 3.7.

There seem to be no strain-rate effects on Young's modulus and Poisson's ratio, as can be seen in Figures 3.42 and 3.43. Average values of Young's modulus and Poisson's ratio of all the specimens included in Tables 3.6 and 3.7 are tabulated in Table 3.9. The second constant slopes of the  $\epsilon_2$  versus  $\epsilon_1$  or  $\epsilon$  curve will be shown in the next chapter.

Table 3.9 Average Dynamic Young's Modulus and Poisson Ratio of WES and SRI Concretes

|                 | WES                    | SRI                    |
|-----------------|------------------------|------------------------|
| Young's Modulus | 41.3 GPa<br>(5990 ksi) | 30.7 GPa<br>(4450 ksi) |
| Poisson Ratio   | 0.263                  | 0.270                  |

### 3.3 Lateral Inertia Estimation in Unconfined Tests

It has been suggested that the enhanced dynamic strength is a consequence of the lateral inertia confinement of the interior. The lateral inertia and the radial stress

induced by it were estimated as follows. Shear stresses were assumed negligible, and the specimen deformation was assumed uniform. Then the radial strain and hoop strain are equal and independent of the distance  $r$  from the centerline and can be determined by the surface measurement of the lateral strain. The radial displacement  $u$  at radial coordinate  $r$  is given by

$$u = r \epsilon_\theta \quad (3.4)$$

which can be substituted into the radial equation of motion,

$$\frac{\partial \sigma_r}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2} \quad (3.5)$$

to give 
$$\frac{\partial \sigma_r}{\partial r} = \rho \ddot{\epsilon}_\theta r \quad (3.6)$$

whence 
$$\sigma_r = -\frac{1}{2} \rho \ddot{\epsilon}_\theta R^2 [1 - (r/R)^2] \quad (3.7)$$

where  $R$  is the outside radius where the surface strains are measured. Equation (3.7) was used to estimate the confining pressure  $|\sigma_r|$  at various values of  $r/R$  for the nine WES and eight SRI strain-gaged specimens. Two regimes were considered: before yield and between yield and maximum stress.

Before Yield the Poisson's ratio was approximately constant, and the second time derivatives of the hoop strain were calculated from the relationship  $\ddot{\epsilon}_\theta = -\nu \ddot{\epsilon}$  where  $\dot{\epsilon}$  is the axial strain rate from the SHPB records, since the SHPB strain rates were directly recorded and were much smoother

than the differentiated hoop strain records. In this regime the lateral inertia confinement values of  $|\sigma_r|$  were estimated in two ways:

- (1) by using the average  $\dot{\epsilon}$  from the start to the point of maximum  $\dot{\epsilon}$ , and
- (2) by using the average  $\dot{\epsilon}$  from the start to the yield point.

The first method gave higher values during the early elastic part of the regime where the strain rate was increasing most rapidly. Both methods should overestimate the actual lateral inertia confinement at the yield point, because the strain acceleration at the yield point is lower than the average values up to that point. Plots of the confining pressure versus  $r/R$ , estimated by the first method for WES and SRI specimens are given in Figures 3.44 and 3.45. The maximum confining pressure occurs at  $r = 0$ , and the maximum value estimated there for the highest impact speed (firing pressure 600 psi) was 1.67 MPa or 243 psi, which is not significant compared to the static unconfined strength (around 101 MPa or 14600 psi) and therefore cannot be considered to be the cause of the enhanced dynamic yield stresses shown in Subsection 3.2.2.

Between Yield and Maximum Stress the slope of the strain rate  $|\dot{\epsilon}|$  versus time curve kept negative or vanished. In other words, the slope,  $\ddot{\epsilon}$ , of the  $\dot{\epsilon}$  versus time curve was always positive or zero. In this region, the apparent

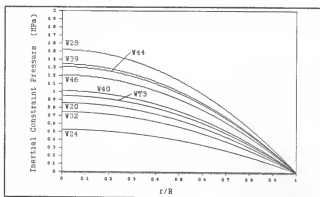


Figure 3.44 Lateral Inertial Confining Pressure Versus  $r/R$  of WES Specimens Based on Average Hoop Strain Acceleration Between Beginning and Time of Maximum Axial Strain Rate

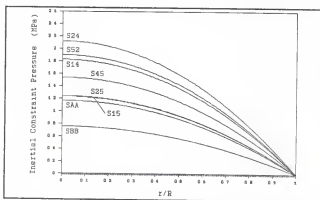


Figure 3.45 Lateral Inertial Confining Pressure Versus  $r/R$  of SRI Specimens Based on Average Hoop Strain Acceleration Between Beginning and Time of Maximum Axial Strain Rate

Poisson's ratio -  $d\epsilon_y/d\epsilon$  was also approximately constant (see Figures 3.35 to 3.40). Then  $\bar{\epsilon}_y$  was always negative or zero. The lateral inertia confining stress  $\sigma_x$  calculated with Equation 3.7 should be a tensile stress or zero.

After maximum stress, analysis based on continuum mechanics would be questionable because of the extensive macroscopic cracking in the specimen.



CHAPTER 4  
DEFORMATION MODEL AND FAILURE CRITERION OF  
CONCRETE UNDER DYNAMIC COMPRESSIVE LOADING

4.1 A Rate-Dependent Model for Concrete  
Under Uniaxial Dynamic Compression

More analysis on the SHPB tests of strain-gaged unconfined WES and SRI specimens is given in this chapter to present a relation between deformation and stress. As analyzed in Section 3.2, WES and SRI concrete specimens showed elastic behavior with an approximately constant Young's modulus and Poisson's ratio below certain stress values, which were defined as the yield stresses and determined by the sudden change in slope of the  $\epsilon_0$  versus  $\epsilon_1$  or  $\epsilon$  curve. Young's modulus and Poisson's ratio appear independent of strain rate in the strain-rate range achieved in the tests of this research (see Table 3.9 for values of Young's modulus  $E$  and Poisson's ratio  $\nu$ ), but the yield stress increases with the average strain rate from the test start to the yield,  $\Delta\epsilon/\Delta t$  (see Equation (3.3) for strain-rate dependence of the yield stress  $\sigma^Y$ ). A rate-dependent model is proposed, in which the strain rate  $\dot{\epsilon}$  is assumed to be composed of two parts, elastic strain rate  $\dot{\epsilon}^e$  and inelastic strain rate  $\dot{\epsilon}^i$ ,

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^i \quad (4.1)$$

for the phase beyond yield but before strain-softening unloading starts. In this model concrete is assumed homogeneous and isotropic.

Elastic Strain Rate  $\dot{\epsilon}^e$  is determined by the stress rate  $\dot{\sigma}$  and Young's modulus as

$$\dot{\epsilon}^e = \dot{\sigma}/E \quad (4.2)$$

Inelastic Strain Rate  $\dot{\epsilon}^i$  was studied for all the strain-gaged unconfined specimens listed in the previous chapter, except for two WES and two SRI specimens where the gage seemed broken before the maximum stress, as follows.

$$\dot{\epsilon}^i = \dot{\epsilon} - \dot{\sigma}/E \quad (4.3)$$

where  $\dot{\epsilon}$  was strain rate of the specimen from the SHPB records,  $E$  was Young's modulus calculated for each individual test (Tables 3.8 and 3.9), and  $\dot{\sigma}$  was calculated from the  $\sigma_2$  versus time records. The finite-difference formula of the second order was used,

$$\dot{\sigma}(t) = \frac{\sigma(t+\Delta t) - \sigma(t-\Delta t)}{2\Delta t} \quad (4.4)$$

where  $\Delta t$  was taken as 0.5 microsecond. Values of stress  $\sigma_2$  and inelastic strain rates at selected inelastic-strain levels (0.0125%, 0.025%, 0.0375%, 0.05% and 0.0625%) before unloading are chosen from the SHPB test records of the seven WES specimens and the eight SRI specimens, and plotted in Figures 4.1 and 4.2 for WES and SRI concrete respectively.

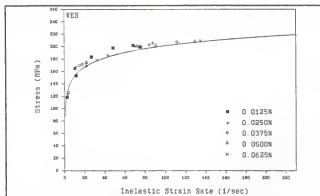


Figure 4.1 Stress Versus Inelastic Strain Rate Between Yield and Unloading for WES Specimens

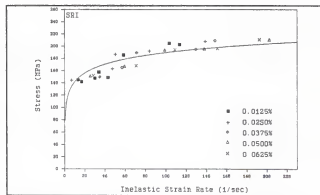


Figure 4.2 Stress Versus Inelastic Strain Rate Between Yield and Unloading for SRI Specimens

Also plotted in the figures are the semilogarithmic regression curves for maximum stress versus strain rate at maximum stress, which were already shown in Figures 3.29 and 3.31. It is seen that points of stress versus inelastic strain rate are very close to the regression for maximum stress versus strain rate at maximum stress. In other words, independent of inelastic strain, stress develops with strain rate along the paths the regression functions describe (Formula (3.1) or (3.2)). Inversely,

$$\dot{\epsilon}^i = \dot{\epsilon}_0 \exp\left(\frac{\sigma - A}{B}\right) \quad (4.5)$$

$$\text{or} \quad \dot{\epsilon}^i = \frac{\dot{\epsilon}_0}{a} \sigma^{1/b} \quad (4.6)$$

where coefficients  $A$ ,  $B$ ,  $a$  and  $b$  for  $\dot{\epsilon}_0 = 1/\text{sec}$  (compression as positive) are listed in Table 3.5.

Since stress in loading develops along a path that the instant inelastic strain rate regulates, points of maximum stress versus strain rate at maximum stress for different specimens at different strain levels, no matter whether the specimen was fractured or not after the test, must be located along this path, because elastic strain rate vanishes at the stress peak and therefore strain rate at maximum stress is pure inelastic strain rate. Though Formulas (3.1) and (3.2) or their inverse forms Formulas (4.5) and (4.6) can be used to describe how maximum stress

Table 4.1 The Apparent Poisson's Ratio  
of WES Specimens after Yield

| I.D. | The Apparent<br>Poisson's Ratio | Ave Strn Rate*<br>(1/sec) |
|------|---------------------------------|---------------------------|
| W24  | 0.347                           | 9.85                      |
| W02  | 0.326                           | 10.4                      |
| W20  | 0.620                           | 22.9                      |
| W40  | 0.636                           | 34.9                      |
| W44  | 0.560                           | 75.2                      |
| W46  | 0.458                           | 98.1                      |
| W39  | 0.588                           | 114                       |
| W28  | 0.627                           | 155                       |

\* Average strain rate, or  $\Delta\epsilon/\Delta t$ , from yield to maximum stress  $\sigma_2$ .

Table 4.2 The Apparent Poisson's Ratio  
of SRI Specimens after Yield

| I.D. | The Apparent<br>Poisson's Ratio | Ave Strn Rate*<br>(1/sec) |
|------|---------------------------------|---------------------------|
| S15  | 0.715                           | 30.9                      |
| S25  | 1.99                            | 48.7                      |
| S45  | 0.304                           | 98.9                      |
| S14  | 0.312                           | 141                       |
| S64  | 1.64                            | 226                       |
| SB   | 2.45                            | 15.0                      |
| SA   | 1.34                            | 134                       |

\* Average strain rate, or  $\Delta\epsilon/\Delta t$ , from yield to maximum stress  $\sigma_2$ .

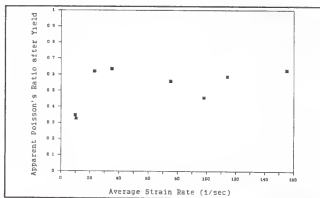


Figure 4.3 Apparent Poisson's Ratio Versus Average Strain Rate of WES Specimens

or strength increases with strain rate, they cannot be used as a failure criterion.

The lateral deformation of the concrete specimen before yield was determined by an approximate Poisson's ratio when the specimen was axially loaded. Tables 4.1 and 4.2 show the apparent Poisson's ratio after yield before maximum stress with the average strain rate from yield to maximum stress, i.e.  $\Delta\epsilon/\Delta t$ . The apparent Poisson's ratio after yield is greater than the elastic Poisson's ratio, though the data for SRI specimens scatter greatly. So far as WES specimens are concerned, values of the apparent Poisson's ratio after yield of six specimens are close (Figure 4.3). The average of them is 0.582. In the other two specimens, W24 and W20, the apparent Poisson's ratio was less than 0.5, which indicated that no dilatation occurred in these tests.

#### 4.2 A Cumulative Failure Criterion for Concrete Under Uniaxial Dynamic Compression

A criterion is desired to indicate ending of the hardening stage or starting of the softening stage, which are identical in the nondelayed fracture tests. The starting of the softening stage draws an application limit of the rate-dependent model proposed above, but the ending of the hardening stage first presents the maximum stress.

Delayed fracture observed in low-speed-impact tests such as W24, where stress remained constant or varied slowly

after it reached maximum value, strongly indicates that crack growth and interaction need time, and stress cannot be a criterion for dynamic fracture of concrete. Even for nondelayed fracture tests where stress turned down sharply as soon as the maximum value of stress was reached, fracture should also be considered a process of crack nucleation and growth. Therefore, the fracture criterion should be of a cumulative type to reflect the process of damage accumulation. Since no observation on cracks inside WES and SRI specimens during the SHPB tests has yet been made, macroscopic quantities recorded in the tests were studied for a failure criterion.

First of all, strain may be considered as a cumulative quantity. As shown in Figures 3.33 and 3.34, critical strain increases with the strain rate at maximum stress but neither strain rate nor its inelastic part remained constant in any test. Therefore, it is difficult to abstract a criterion of fracture based on the critical strain from the test data.

The cumulative work done on the specimen was calculated to see if it would provide a criterion. The loading work per unit volume till yield (elastic strain energy density at yield) is designated as  $W^e$ , the loading work per unit volume from yield to the beginning of maximum stress is designated as  $W^{y1}$ . When there is an approximately constant maximum stress region, the loading work per volume from the



Table 4.3 Work per Unit Volume and  $K^c$  of WES Specimens

| I.D. | $W^*$<br>(kPa) | $W^{*1}$<br>(kPa) | $W^{*2}$<br>(kPa) | $W^{i1}$<br>(kPa) | $W^{i2}$<br>(kPa) | $K^c$<br>(Pa·sec) | Ave Strn Rate*<br>(1/sec) |
|------|----------------|-------------------|-------------------|-------------------|-------------------|-------------------|---------------------------|
| W24  | 167            | 69.7              | 77.3              | 59.8              | 71.2              | 539               | 9.85                      |
| W20  | 255            | 143               | 75.0              | 61.7              | 101               | 502               | 22.9                      |
| W40  | 227            | 164               | 89.0              | 73.5              | 71.5              | 510               | 34.9                      |
| W44  | 273            | 230               | 0                 | 98.6              | 0                 | 486               | 75.2                      |
| W46  | 294            | 279               | 0                 | 91.3              | 0                 | 491               | 98.1                      |
| W39  | 323            | 303               | 0                 | 131               | 0                 | 500               | 114                       |
| W28  | 320            | 411               | 0                 | 169               | 0                 | 530               | 155                       |

\* Average strain rate, or  $\Delta\epsilon/\Delta t$ , from yield to the beginning of maximum stress  $\sigma_2$ . The average strain rate from the beginning to the end of the maximum stress is about the same as the strain rate at the maximum stress, shown in Table 3.2.

Table 4.4 Work per Unit Volume and  $K^c$  of SRI Specimens

| I.D. | $W^*$<br>(kPa) | $W^{*1}$<br>(kPa) | $W^{*2}$<br>(kPa) | $W^{i1}$<br>(kPa) | $W^{i2}$<br>(kPa) | $K^c$<br>(Pa·sec) | Ave Strn Rate*<br>(1/sec) |
|------|----------------|-------------------|-------------------|-------------------|-------------------|-------------------|---------------------------|
| S15  | 320            | 104               | 69.9              | 35.1              | 82.3              | 410               | 30.9                      |
| S25  | 355            | 239               | 66.0              | 143               | 72.0              | 422               | 48.7                      |
| S45  | 275            | 265               | 45.0              | 94.1              | 51.9              | 432               | 98.9                      |
| S14  | 365            | 332               | 0                 | 146               | 0                 | 314               | 141                       |
| S52  | 363            | 462               | 0                 | 216               | 0                 | 435               | 171                       |
| S24  | 479            | 543               | 0                 | 283               | 0                 | 359               | 226                       |
| SB   | 217            | 91.5              | 47.7              | 27.3              | 55.0              | 389               | 15.0                      |
| SA   | 352            | 341               | 0                 | 37.8              | 0                 | 538               | 134                       |

\* Average strain rate, or  $\Delta\epsilon/\Delta t$ , from yield to the beginning of maximum stress  $\sigma_2$ . The average strain rate from the beginning to the end of the maximum stress is about the same as the strain rate at the maximum stress, shown in Table 3.2.

beginning of maximum stress to the end of maximum stress is designated as  $W^{y2}$ . The dissipated part of  $W^{y1}$  and  $W^{y2}$ , denoted as  $W^{i1}$  and  $W^{i2}$ , are calculated and tabulated in Tables 4.3 and 4.4 for all the WES and SRI specimens listed in Tables 4.1 and 4.2. The following are the mathematical definitions of  $W^*$ ,  $W^{y1}$ ,  $W^{y2}$ ,  $W^{i1}$ , and  $W^{i2}$ .

$$W^* = \int_{\text{start}}^{\text{yield}} \sigma_2 d\epsilon \quad (4.7)$$

$$W^{y1} = \int_{\text{yield}}^{\text{max } \sigma_2 \text{ begins}} \sigma_2 d\epsilon \quad (4.8)$$

$$W^{y2} = \int_{\text{max } \sigma_2 \text{ begins}}^{\text{max } \sigma_2 \text{ ends}} \sigma_2 d\epsilon \quad (4.9)$$

$$W^{i1} = \int_{\text{yield}}^{\text{max } \sigma_2 \text{ begins}} \sigma_2 d\epsilon^i \quad (4.10)$$

$$\text{and } W^{i2} = \int_{\text{max } \sigma_2 \text{ begins}}^{\text{max } \sigma_2 \text{ ends}} \sigma_2 d\epsilon^i \quad (4.11)$$

Obviously,  $W^{y2}$  and  $W^{i2}$  are both zero for non-delayed fracture tests.  $W^{y1}$  is the sum of  $W^{i1}$  and the elastic strain energy density the specimen absorbs from yield to the beginning of maximum stress. All the work done during the maximum-stress plateau was  $W^{y2}$ . Calculated  $W^{y2}$  and  $W^{i2}$  were not identical because stress during the maximum-stress plateau in the delayed-fracture tests was not exactly constant. According to Tables 4.3 and 4.4, the dissipated work per unit volume needed for fracture,  $W^{i1}$ , increased with the average strain rate  $\Delta\epsilon/\Delta t$  from yield to maximum

stress. Among delayed-fracture tests,  $W^{y2}$  and  $W^{i2}$  did not show increase with strain rate. Neither  $W^{y1} = \text{constant}$  nor  $W^{i1} = \text{constant}$  can be taken as a criterion for predicting the beginning of maximum stress, because the values listed in Tables 4.3 and 4.4 differ significantly for the different tests. There is some consistency of the  $W^{y2}$  values for different specimens of the same concrete, so that  $W^{y2} = \text{constant}$  might be taken as a criterion for predicting the time duration of the maximum stress plateau in the delayed-failure cases.

A possible cumulative criterion for predicting the time at which the maximum stress is reached is here suggested in terms of a damage parameter defined by

$$K = \int_{\text{yield instant}}^t (\sigma - \sigma^y) dt \quad (4.12)$$

where  $\sigma^y$  is the yield stress. When the damage parameter  $K$  is increased up to a critical value  $K^0$ , the maximum stress is reached. Values of the integral (4.12) calculated with recorded  $\sigma_2$  as  $\sigma$  in the formula from the yield instant through the instant of the maximum  $\sigma_2$  for eight WES and seven SRI specimens are shown in Tables 4.3 and 4.4. The  $K^0$  values from different specimens of the same kind of concrete are considerably consistent, even though SB and SA were 3 inches long, and the rest of the SRI specimens in Table 4.4 were 1.5 inches long. The critical damage parameter  $K^0$  is

taken as the average of the experimental values, 508 Pa·sec for WES concrete and 412 Pa·sec for SRI concrete.

This suggested criterion (4.12) is similar to a failure criterion proposed for tensile loading by Tuler and Butcher (1968, after Zukas, 1982) for computation codes in terms of a damage parameter  $K$  defined by

$$K = \int_0^t (\sigma - \sigma_0)^\lambda dt \quad (4.13)$$

where  $\sigma_0$  is a threshold stress level below which no significant damage will occur regardless of stress duration, and  $\lambda$  is considered a material parameter chosen to fit experimental data. Failure is assumed to occur instantaneously when a critical value,  $K^c$ , of the damage parameter  $K$  is reached.

Formula (4.12) is a case specific for the concretes with  $\lambda=1$ . It is assumed that concrete is linearly elastic below the yield point, so that no significant damage occurs until stress exceeds the yield stress. Yield stress  $\sigma^y$  is taken as a threshold stress level in Formula (4.12). Stresses  $\sigma$  and  $\sigma^y$  in Formula (4.12) are both strain-rate dependent so that the criterion is a strain-rate dependent cumulative criterion. Though this criterion does not predict whether the fracture will be delayed or catastrophic, the damage accumulation function  $K$  characterizes reduction of stiffness of concrete caused by damage. The criterion (4.12) may be called a failure-

initiation criterion, because, when  $K$  approaches  $K^c$ , the concrete has accumulated such damage that it cannot bear higher compressive stress later. As a material parameter,  $K^c$  characterizes capability of concrete to resist compressive loading under the unconfined condition, that is, the limit stress level concrete can sustain after being damaged under varying high strain rate. However, the process of crack development during the delayed time has not been disclosed. Some questions remain open, for example, how a specimen which is unloaded before the end of a delayed-fracture stress plateau will react to a second loading, and whether  $K^c$  is independent of lateral dimensions of the specimen though it may have nothing to do with the specimen thickness. More tests on specimens of different sizes with incident pulses of different durations as well as crack observation would be useful.

## CHAPTER 5 CRACK OBSERVATION OF IMPACTED CONCRETE SPECIMENS

### 5.1 Introduction

The objectives of the research reported here were to develop procedures and demonstrate the feasibility of using them to make micrographic examinations of undamaged and damaged SHPB compression specimens to determine crack development characteristics in the impacted specimens. Several previously proposed models for concrete failure under impact involve crack initiation, propagation and coalescence assumptions, which had so far not been verified by observations of the crack patterns after varying amounts of damage before general failure. The results of such observations could provide a qualitative understanding of the physical mechanism leading to failure and potentially a quantitative basis for modeling.

In order to accomplish the objectives two experimental procedures had to be developed. One was the petrographic procedure with which the specimen crack pattern in a longitudinal slice was stabilized by furfuryl alcohol infiltration and polymerization to fill the cracks with a rigid furan resin before polishing for micrographic

examination. This procedure was suggested by Professor Richard Connell of the Materials Science and Engineering Department, University of Florida and performed by Miss Kim Gudmundson, who was an undergraduate in that department. To recover specimens with various amounts of deformation before complete failure, a steel collar was placed around each specimen, large enough that it did not provide lateral confinement. Specimens were cut slightly longer than the collar, ends ground to assure flatness and parallelism and to produce a range of excess lengths from 0.0012 to 0.012 times the collar length. Thus after an axial strain of 0.0012 to 0.012 the steel collar began to stop the loading. Similar collar tests were performed quasistatically at comparable levels of axial deformation.

## 5.2 Concrete Specimens and Experimental Techniques

### 5.2.1 Concrete Specimens and Test Procedures

Test material was cast in split tube molds with inside diameters of 2 inches (50.8 mm) and lengths of approximately 5 inches (127 mm). After curing, two test specimens 1.75 inches (44.5 mm) long were sawed from the central regions of the resulting cylinders. The ends of the specimens were then ground flat and parallel on a surface grinder to produce a series of finished lengths ranging from 1.709 to

1.727 inches (43.4 to 43.8 mm) for subsequent static and dynamic testing.

A steel collar 1.707 inches (43.36 mm) in length with a bore of 2.050 inches (52.1 mm) and an outside diameter of 3 inches (76.2 mm) was fabricated. The collar, which is shown in Figure 5.1 (not to scale), stopped each static and dynamic test near the desired level of strain.

The concrete specimens used for this group of tests were prepared in the Civil Engineering Department, University of Florida with the assistance of Mr. Daniel Richardson. The concrete mix is shown in Table 5.1. The average density of the specimens was 2.22 g/cm<sup>3</sup>. Unconfined static compressive strength was around 69 MPa (10 KSI).

Table 5.1 Concrete Mixture Design

|                                  |          |
|----------------------------------|----------|
| Type II Portland Cement          | 32.7 lb. |
| Brooksville Aggregate No. 89 *   | 94.2 lb. |
| Sand (Keuka, FL, Pit No. 76-137) | 60.4 lb. |
| Water                            | 16.5 lb. |
| Water/Cement Ratio               | 0.50     |
| Slump                            | 2.25 in. |

\* The Brooksville, Florida, No. 89 is a mixture of coarse and fine manufactured Florida limestone aggregate with a maximum size of 3/8 inch (9.5 mm).

Two series of tests were conducted. The series done in the SHPB will be called dynamic. The six dynamic test specimens were designed for predicted strains of 0.0029,



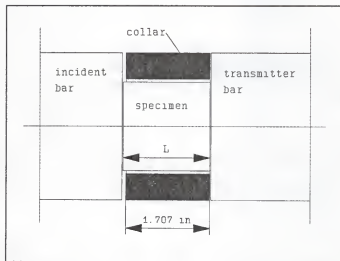


Figure 5.1 Collar Used to Interrupt Tests

0.0047, 0.0064, 0.0081, 0.0099 and 0.0116 before the collar began to act. In each of the dynamic tests some further axial deformation of the specimen occurred along with the axial elastic shortening of the collar, so that the measured maximum strains (corresponding to the maximum crack development) were larger than the predicted strains by approximately 0.002.

The second series of tests used a hydraulic press to slowly apply the load and will be referred to as static. Once again, the steel collar was used to arrest the test at a given level of strain. Petrographic examination was made of six specimens with strain levels over a range comparable to that of the six dynamic tests, although the strains in the series are not exactly matched.

#### 5.2.2 Petrographic Examination Procedures

After each specimen was tested, it was stabilized by potting it in a polyester mounting resin (LECO Castolite), which helped keep the specimen together throughout subsequent handling. After curing of the polyester, a longitudinal slice approximately 0.060 inch (1.52 mm) thick was cut from the center of each specimen.

In order to further stabilize the structure during mechanical polishing and to provide visual contrast to aid in examination of the crack patterns, a procedure based on the vacuum infiltration and polymerization of furfuryl

alcohol [ $2-(C_4H_5O)CH_2OH$ ] was used. After infiltration with furfuryl alcohol, about 20 drops of concentrated hydrochloric acid were carefully added to the approximately 50 cc of alcohol, initiating a polymerization reaction which causes the amber colored furfuryl alcohol to become a dark brown rigid furan resin within the existing cracks, which reinforced the structure and inhibited further crack development during polishing.

Mounted specimens were ground flat by hand on a succession of silicon carbide grinding papers (180 - 320 - 600 grit), using no lubricant. Finally, each was polished using 6 micron diamond paste on a synthetic fiber polishing wheel, using kerosene as a lubricant.

A macrophotograph was made of each mounted section for later use in marking the paths of cracks. Large cracks could be readily seen in the photographs, but smaller cracks can only be observed by using the metallographic microscope at 5X to 50X. To generate an overall view of the emerging crack patterns, a routine was developed wherein the entire surface of the section was scanned using the microscope and any cracks that were found were marked on a large black and white print of the section macrophotograph using a blue pen. Tracings on plain white paper were made to isolate and enhance the crack patterns.

To deduce the length of cracks per unit area in the cross section and the surface area of cracks per unit

volume, the technique of line intercept counting was employed. A grid of sample lines was laid over the traced crack pattern, and the number of intercepts each sample line made with cracks was recorded. The total number of intercepts, divided by the total length of sample line, can be shown to be equal to  $2/\pi$  times the crack length per unit area. The crack length per unit area can further be shown to equal  $\pi/2$  times the total crack surface area per unit volume (counting both surfaces created by the crack), so that the intercept count per unit length gives the crack surface area per unit volume directly; see Underwood (1968).

### 5.3 Test Results

#### 5.3.1 Results of Dynamic Tests

The mechanical results of the dynamic tests terminated by the collar will be illustrated by two figures to explain how the measurements were made and then summarized in Table 5.2. All the collar tests used the same gas-gun firing pressure and striker-bar impact speed as the test without a collar. Ideally the stresses versus time and the stress-strain curves would have the same appearance as the curves for the no-collar specimen until the strain had reached the "predicted strain" value that just reduced the specimen length to the collar length.

The actual results were not so ideal. Even after dispersion correction both Stress  $\sigma_1$  and Stress  $\sigma_2$  versus time still showed oscillations believed related to the three dimensionality of the wave fronts, induced by the area mismatch between bars and specimen, and possibly to the dynamics of the loose collar, and the two stress plots did not usually reach agreement before the collar took over. Figure 5.2 shows Stress  $\sigma_1$ , Stress  $\sigma_2$ , Strain Rate  $\dot{\epsilon}$ , and Strain  $\epsilon$  versus time for a specimen with predicted strain of 0.0099 for collar contact (approximately 1 percent strain or 100 on the ordinate scale). Note the three definitions of the ordinate scale. With strain, for example, a reading of 100 is to be multiplied by 1/10,000 to give 0.01 or 1 percent strain.

This strain is reached at time = 129 microsec, marked by a square on the strain curve. At this point both the recorded Stress  $\sigma_1$  and Stress  $\sigma_2$  (also marked by squares) are well above the approximately 100 MPa failure stress of the no-collar test (which occurred at a strain of 0.0064 in the no-collar test). The stress records were not considered reliable for determining collar action. The strain record proved more useful.

The strain record shows a maximum strain plateau of about 0.0128 during the collar action, falling to about 0.0105 at the end of the plot, also marked by a square, after the elastic compression of the collar had been

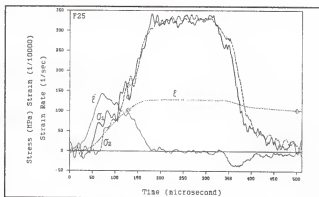


Figure 5.2 Stresses, Strain and Strain Rate Versus Time for Collar Test of Specimen F25

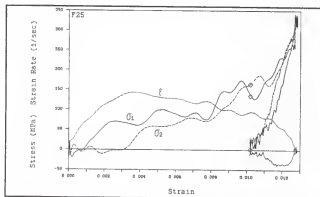


Figure 5.3 Stresses and Strain Rate Versus Strain for Collar Test of Specimen F25

unloaded. The final value of strain after unloading is more easily read on the stress-strain curves of Figure 5.3 as about 0.0102. This final unloaded value of strain is very close to the predicted strain at collar contact. The indicated stress-strain curves of Figure 5.3 indicate that both stresses had surpassed the no-collar test strength before this strain was reached. It is clear that by the time the predicted strain of about 0.01 had been reached the collar was already carrying a considerable part of the load.

The strain results summarized in Table 5.2 show a remarkable consistency. They will be used to correlate with the crack distribution data.

Figure 5.4 shows the Stress  $\sigma_2$  versus strain curve of a no-collar dynamic test. The six points marked on it are at the maximum strains of Table 5.2, indicating what points of

Table 5.2 Predicted Strain at Collar Contact and Measured Residual and Maximum Strain

| SPECIMEN<br>No. | PREDICTED<br>Strain | RESIDUAL<br>Strain | MAXIMUM<br>Strain |
|-----------------|---------------------|--------------------|-------------------|
| F21             | 0.0029              | 0.0023             | 0.0041            |
| F22             | 0.0047              | 0.0046             | 0.0065            |
| F23             | 0.0064              | 0.0068             | 0.0092            |
| F24             | 0.0081              | 0.0082             | 0.0103            |
| F25             | 0.0091              | 0.0102             | 0.0128            |
| F04             | 0.0116              | 0.0106             | 0.0126            |

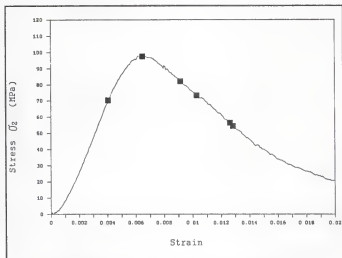


Figure 5.4 Dynamic Stress-Strain Curve for No-Collar Test. Points Marked Correspond to Maximum Strain in Collar-Interrupted Tests



the unconfined dynamic stress-strain curve were reached before unloading. These points represent approximately the damage conditions for the crack patterns of the dynamic before unloading. These points represent approximately the damage conditions for the crack patterns of the dynamic collar tests to be reported. Apparently the first one is at about two-thirds of the failure strain, the second one is just at the failure strain, and the other four are from the strain softening regime. This interpretation takes no account of likely random differences in the mechanical properties of the various test specimens.

#### 5.3.2 Crack Patterns and Crack Surface Area

Figures 5.5 and 5.6 show typical crack patterns for one example of each of the two test series. When all 12 patterns are compared, differences between the dynamic series and the static series are apparent. In the dynamic series, the cracks are more uniformly distributed, while the cracks in the static series specimens tend to be concentrated along definite bands between the center and the surface of the specimen.

The results of the crack surface area measurements, as derived from the line intercept counting on each section, are presented in Figure 5.7. The surface area per unit volume is plotted against the observed maximum strain. Table 3 lists the data on which Figure 5.7 is based.

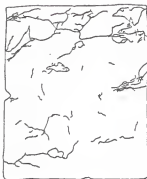


Figure 5.5 Crack Pattern Tracing of Dynamic Test Specimen with Maximum Strain of 0.0041



Figure 5.6 Crack Pattern Tracing of Static Test Specimen with Maximum Strain of 0.0049

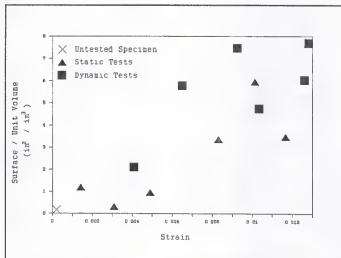


Figure 5.7 Crack Surface Area per Unit Volume Versus Maximum Strain

Table 5.3 Crack Surface Area per Unit Volume  
at Various Strain Levels

Dynamic Test Results

| Specimen No. | Predicted Strain | Maximum Strain | Surface/Unit Vol. ( $\text{in}^2/\text{in}^3$ ) |
|--------------|------------------|----------------|-------------------------------------------------|
| F21          | 0.0029           | 0.0041         | 2.14                                            |
| F22          | 0.0047           | 0.0065         | 5.78                                            |
| F23          | 0.0064           | 0.0092         | 7.46                                            |
| F24          | 0.0081           | 0.0103         | 4.76                                            |
| F25          | 0.0099           | 0.0128         | 7.68                                            |
| F04          | 0.0116           | 0.0126         | 6.06                                            |

Static Test Results

| Specimen No. | Estimated Predicted Strain | Maximum Strain | Surface/Unit Vol. ( $\text{in}^2/\text{in}^3$ ) |
|--------------|----------------------------|----------------|-------------------------------------------------|
| F51          | 0.0012                     | 0.0014         | 1.16                                            |
| F52          | 0.0029                     | 0.0031         | 0.28                                            |
| F53          | 0.0047                     | 0.0049         | 0.94                                            |
| F54          | 0.0081                     | 0.0083         | 3.34                                            |
| F55          | 0.0099                     | 0.0101         | 5.92                                            |
| F56          | 0.0116                     | 0.0118         | 3.46                                            |

Untested Specimen

| Specimen No. | Surface/Unit Volume ( $\text{in}^2/\text{in}^3$ ) |
|--------------|---------------------------------------------------|
| F50          | 0.2                                               |

For maximum strains up to a threshold strain of about 0.006 (near the failure strains in no-collar tests) there is a moderate amount of cracking. Above a maximum strain of 0.006 (corresponding to the strain-softening regime in the no-collar tests) there is a significantly greater crack development. From the limited data an upward trend of crack density with strain cannot be confirmed in the regime below the threshold or in the regime above the threshold. For the strain-softening regime, this may mean that once the major cracks have formed, much of the deformation continues by sliding on the existing cracks. For the lower-strain regime, perhaps a closer examination both of the microcracks not seen by the optical microscope and of more sections from each specimen would be informative.

The very small crack density in the section from the untested specimen confirms that the sectioning, infiltration and polishing operations do not introduce any significant cracking, so that the crack densities seen in the examination slices may be considered to represent the condition in the test-damaged specimens before sectioning.

## CHAPTER 6 AN ELASTOVISCOPLASTIC MODEL FOR CONCRETE

### 6.1 Background

#### 6.1.1 Introduction

The purpose of this chapter is to propose an elastoviscoplastic model for high-strength concrete in the three-dimensional stress case. Reviewed in this section are some previous plastic and viscoplastic models, to which the proposed model is closely related.

A number of types of viscoplastic models are based on the concept of overstress, the difference between the current stress state and the corresponding stress state on the static yield surface. Therefore, in order to construct a viscoplastic model of one of those types, it is necessary to set up the static yield surface first. The employment of an overstress or an overstress-like type of model offers convenience in taking advantage of research results in rate-independent plasticity, which often provides an idealization for behavior of a certain rate-sensitive material before sufficient data are acquired to make mathematical analysis concerning rate-dependence possible. This is the case for concrete. Though rather more static experiments and further

theoretical study with the time-independence assumption are still needed for concrete, abundant knowledge has been accumulated without consideration of rate dependence. In this research, an extension of Malvern's overstress equation for the one-dimensional case, i.e., simple tension or compression (1951), whose form is similar to but somewhat different from Perzyna's type (Perzyna, 1963, 1966), is employed as the first step of viscoplastic modeling.

#### 6.1.2 Failure Surface or Perfect-Yield Surface

A simple failure model for concrete could be the Drucker-Prager model (Drucker and Prager, 1951), a Mises type generalization of the Coulomb failure criterion, which was originally proposed for soil. The Drucker-Prager model is a cone-shaped surface in the principal stress space described as

$$\alpha I_1 + \sqrt{J_2} - \kappa = 0 \quad (6.1)$$

where  $I_1$  is the first invariant of the stress tensor,  $J_2$ , the second invariant of the deviatoric stress tensor, and  $\alpha$  and  $\kappa$ , material constants. The Drucker-Prager surface (6.1) has been taken as a fixed yield surface in elastic and perfect-plastic analysis in conventional incremental plasticity theory, as well as a failure criterion for elastic analysis. There are basically two shortcomings of the Drucker-Prager criterion in connection with concrete modeling: the linear relation between  $I_1$  and  $\sqrt{J_2}$ , and the

circular cross section of the conical surface. To overcome these two shortcomings, more models for concrete have arisen. The major efforts to make the yield surface coincide better with experimental data of failure stresses have been to include more stress invariants with an increasing number of material constants or parameters. Bresler and Pister (1958), Willam and Warnke (1974), Argyris et al. (1974), and Lade (1982) suggested three-parameter models. Ottosen (1977), Reimann in 1965 (according to Chen, 1982) and Hsieh et al. (1982) proposed four-parameter models. Willam and Warnke (1974) presented a five-parameter model. The Willam-Warnke five-parameter model, which will be simply called Willam-Warnke model in the following for brevity, has been commented on as the most appropriate model for defining the shape of a triaxial failure surface (Chen, 1984). Podgórski (1983) proposed a general form for failure surfaces of isotropic media. Its application to concrete also contains five parameters.

The Willam-Warnke model includes three stress invariants. In addition to  $I_1$  and  $\sqrt{J_2}$ , an azimuth angle,  $\theta$ , in the deviatoric plane is involved in the model. This angle, which is often called the angle of similarity, will be illustrated in Figure 6.1. and the relation among  $J_2$ ,  $J_3$  and  $\theta$  will be shown in Formula (6.14.c). The Willam-Warnke model is described by its traces in two meridian planes  $\theta = 0$  and  $\theta = \pi/3$  and in planes parallel to the deviatoric



plane in principal-stress space. In this model, the traces in the meridian planes are quadratic parabolas rather than the straight lines of the Drucker-Prager model, and the traces in the planes parallel to the deviatoric plane look like bulged triangles rather than the circles of the Drucker-Prager model. In the model to be proposed, the Willam-Warnke surface is employed; therefore the Willam-Warnke model will be described in detail later.

### 6.1.3 Strain-Hardening Cap

It has been observed that many geological materials experience plastic deformation. In other words, if the material is unloaded from a given state during loading, it may not return to its original configuration, and it experiences inelastic or plastic deformations. Many experimental investigators have shown the quasistatic hardening behavior of concrete (e.g., see Scavuzzo et al. 1983). As the earliest effort to describe the strain hardening behavior of geological materials, Drucker et al. (1955) proposed successive yield surfaces to use with the Drucker-Prager failure surface. The yield surface that defines the hardening behavior is often called *the hardening cap*. Drucker et al. assumed, as an approximation, the hardening cap to be a spherical surface for simplicity. Different investigators have since proposed different shapes of the cap.

One commonly used cap model is the one proposed by DiMaggio and Sandler (1971) in which there is a fixed portion (failure surface or perfect-yield surface) fitted with a hardening cap. The deviatoric traces of either the fixed portion or the cap are circles. This model has been applied to describing behavior of concrete and rocks in recent years. As they proposed, the meridian of the cap is elliptical. To specify the size and location of the cap two parameters are needed. They are the half axis along the hydrostatic axis and the distance of the center of the ellipse from the origin. The half axis in the direction of  $\sqrt{2}J_2$  is determined by the two parameters and the failure-surface meridian, since the highest point of the elliptical cap in the direction of  $\sqrt{2}J_2$  is just the intersection of the meridians of the failure surface and of the cap.

Faruque (1987) proposed for concrete that the cap passes through the origin to reduce one parameter, which will be shown in Figure 6.4. Also, he suggested that the cross-sections of either the failure surface or the cap in the deviatoric planes are not circles but bulged triangles and thus the stress invariants  $I_1$ ,  $\sqrt{2}J_2$  and  $\theta$  are involved in his model. The failure-surface form he used is analogous to those of Ottosen (1977) and Podgórski (1985). It seems not as mathematically easy to use as the Willam-Warnke model, though the complicated mathematical form does not present more physical meaning. An elliptical hardening cap is

constructed inside the Willam-Warnke surface in the model to be proposed in Section 6.2, and where Faruque's idea is employed so that the cap is suggested to pass through the origin and then is completely defined by its intersection with the failure surface.

#### 6.1.4 Overstress-Type Viscoplastic Model

The early overstress model was proposed by Sokolovskii in 1948 and Malvern (1951) independently. The general form of their overstress function can be expressed as

$$\dot{\epsilon}^p = F[\sigma - g(\epsilon), \epsilon] \quad \text{for } \sigma \geq g(\epsilon) \quad (6.2)$$

where  $F$  may be an arbitrary function with  $F[0, \epsilon] = 0$ , and  $\sigma = g(\epsilon)$  is the quasistatic stress-strain curve. According to this equation, plastic strain rate depends on strain and on overstress, that is on the amount by which the current stress exceeds the stress on the quasistatic stress-strain curve at the same strain. Perzyna (1963) proposed a generalization to three-dimensional multiaxial stress states and has published several more advanced formulations since then. One of these formulation (Perzyna, 1966) is based on the von Mises yield criterion,

$$\sqrt{J_2} = \kappa(W_p) \quad (6.3)$$

where  $J_2$  is the second invariant of the deviatoric stress and isotropic hardening is implied by the dependence on the plastic work  $W_p$ . The formulation gives the deviatoric strain rate as

$$\dot{e}_{ij} = \frac{\dot{s}_{ij}}{2\mu} + \gamma \langle \Phi \left( \frac{\sqrt{J_2}}{\kappa(W_p)} - 1 \right) \rangle \frac{s_{ij}}{\sqrt{J_2}}, \quad (6.4)$$

where  $\langle \Phi(F) \rangle = 0$  for  $F \leq 0$  and  $\langle \Phi(F) \rangle = \Phi(F)$  for  $F > 0$ ;  $s_{ij}$  are the deviatoric stresses;  $\mu$  is the shear modulus; and  $\gamma$ , the viscosity parameter. Perzyna considered it as an extension of Malvern's model for uniaxial stress but Malvern (1984) pointed out that his own formulation is slightly different from Perzyna's. A possible three dimensional version of Malvern's model for uniaxial stress is

$$\dot{e}_{ij} = \frac{\dot{s}_{ij}}{2\mu} + \gamma \langle \Phi \left( \frac{\sqrt{J_2} - \kappa(\bar{\epsilon})}{\kappa_0} \right) \rangle \frac{s_{ij}}{\sqrt{J_2}}, \quad (6.5)$$

where  $\kappa_0$  is a constant and where the isotropic hardening has been expressed by  $\kappa$ , a function of the equivalent plastic strain  $\bar{\epsilon}$ . Perzyna (1966) also applied the concept of overstress to the Drucker-Prager surface as a viscoplastic model for soil, in which overstress is the amount by which the current value of  $\sqrt{(2J_2)}$  exceeds the value of  $\sqrt{(2J_2)}$  on the Drucker-Prager surface at the same hydrostatic pressure, and normality of inelastic strain increments to the potential surface  $\sqrt{(2J_2)} = \text{constant}$  through the current stress point in stress space is taken for granted. It is implicitly suggested that the material of this model fails in a shear mode.

Perhaps two of the most popular viscoplastic formulations are the endochronic theory pioneered by Valanis (1971) and the overstress viscoplastic theory presented by Perzyna. Other viscoplastic formulations include developments by Phillips and Wu (1973), Bodner and Partom (1975), and Katona (1980). Katona and Mulert (1984, p.336) commented on the advantages of Perzyna's viscoplastic theory as they stated reasons of adopting it in their research as follows.

Motivations of adopting Perzyna's elastic/viscoplastic theory in this study are: (1) the formulation is well accepted and well used, (2) the incorporation of the inviscid cap model (or any plasticity yield function) is relatively straightforward, (3) the generality of the time-rate flow rule offers the capability of simulating time-dependent material behavior over a wide range of loading, (4) techniques for parameter identification are feasible, and (5) the formulation is readily adaptable to a numerical algorithm suitable for a finite element procedure.

## 6.2 An Elastoplastic Model for High-Strength Concrete

### 6.2.1 Introduction

An elastoplastic model is proposed in this section and an elastoviscoplastic model based on the elastoplastic model will be presented in the next section. Concrete is treated as homogeneous and isotropic in the proposed models. The elastoplastic model consists of a perfect-yield surface and a strain hardening cap. The Willam-Warnke five-parameter model is employed as the perfect-yield surface.

### 6.2.2 Fundamentals of Plasticity

In almost all rate-independent theories of continuum plasticity (Malvern, 1969), the yield criterion is assumed to depend on the state of stress. A yield function  $f(T)$  is postulated to exist such that the material is elastic for

$$f(T) < 0, \text{ or for } f(T) = 0 \text{ and } \frac{\partial f}{\partial T_{ij}} \dot{T}_{ij} < 0 \quad (6.6.a)$$

$$\text{and plastic for } f(T) = 0 \text{ and } \frac{\partial f}{\partial T_{ij}} \dot{T}_{ij} \geq 0 \quad (6.6.b)$$

where  $T$  is stress tensor and  $f(T) \leq 0$  always. The yield condition of (6.6) can be interpreted geometrically in terms of  $f(T) = 0$ , i.e., a hypersurface in a nine-dimensional Euclidean space in which the stress  $T_{ij}$  are interpreted as nine rectangular Cartesian coordinates. If the material is isotropic, it does not have any preferred directions. Therefore, the yield condition can be expressed only in terms of a function of principal stresses,

$$f(T_1, T_2, T_3) = 0 \quad (6.7)$$

or in terms of a function of the invariants of the stress tensor,

$$f(I_1, I_2, I_3) = 0. \quad (6.8)$$

Obviously, the yield condition for the isotropic material can also be expressed in terms of a function of any independent three of the invariants of the stress tensor and the deviatoric stress tensor, for example,

$$f(I_1, J_2, J_3) = 0 \quad (6.9)$$

$$\text{or } f(I_1, \sqrt{J_2}, \theta) = 0 \quad (6.10)$$

where  $J_2$  and  $J_3$  are the second and the third invariants of the deviatoric stress tensor, and  $\theta$  is the angle of similarity, which will be defined in terms of  $J_2$  and  $J_3$  in Equation (6.14.c).

In the theory of plasticity (Hill, 1950), the direction of the plastic strain increment vector in the strain space, where the  $\epsilon_{ij}^p$ -axes coincide with the  $T_{ij}$ -axes of the stress space, is defined through a flow rule by assuming the existence of a plastic potential function, to which the incremental strain vectors are orthogonal. Then the increments of the plastic strain can be expressed as

$$d\epsilon_{ij}^p = \lambda \frac{\partial Q}{\partial T_{ij}} \quad (6.11)$$

where  $Q$  is the plastic potential function, and  $\lambda$  is a positive scalar factor of proportionality. For some materials, the plastic potential function,  $Q$ , and the yield function,  $f$ , can be assumed to be the same. Such materials are considered to follow the *associative flow rule* of plasticity. The materials for which the yield function  $f$  and the plastic potential function  $Q$  are different are defined to follow *nonassociative flow rules* of plasticity. To ensure an appropriate description of the physical process involved in plastic deformations, Prager (1949) formulated

some assumptions. One of these is, as a result of the assumption of rate independence, the *linearity* assumption,

$$d\epsilon_{ij}^p = D_{ijkl}dT_{kl}, \quad (6.12)$$

where  $D$  is a fourth order plastic compliance tensor. The second assumption is the *condition of continuity*, to guarantee which the neutral loading does not cause plastic deformation. The third assumption is the *condition of uniqueness*. This condition ensures that for a given mechanical state of a body and a system of infinitesimal increments of surface traction, the resulting increments of stresses and strains (elastic and plastic) are unique. During plastic loading, the material remains at yield as it moves from one yield surface  $f(T) = 0$  to another,  $f(T+dT) = 0$ . When this requirement is met, the *condition of consistency* is said to be satisfied,

$$df(T) = 0 \quad (6.13)$$

Drucker (1950, 1951, 1956, 1958) systematized the plasticity theory by his stability postulates. According to him, a stable work-hardening material is defined to be one such that (1) the plastic work done by external agency during the application of the additional stresses is positive, and (2) the net total work performed by the external agency during the cycle of adding and removing stresses is nonnegative. With the assumption of existence of the yield surface and Drucker's definition of stable



plastic material, a series of important results followed from strict mathematical proof, among which are *convexity* of the yield surface, the *associative flow rule*, and *uniqueness* of the plasticity boundary-value problem, so that the fragmentary assumptions proposed before the stability postulates merged to an integral theory.

Drucker (1988) reviewed the recent developments in conventional and unconventional plasticity modeling, in which references are included for more than the fundamentals discussed above.

### 6.2.3 The Willam-Warnke Five-Parameter Surface

The  $(T_1, T_2, T_3)$  coordinate system has been introduced to represent a stress space called principal stress space or the *Haigh-Westergaard stress space*. The yield condition (6.7) for isotropic materials can be geometrically expressed in the space. Every point P in the space having coordinates  $T_1, T_2$ , and  $T_3$  is a possible stress state (Figure 6.1.a). The stress vector OP can be decomposed into two components, the component ON in the direction of the hydrostatic-stress axis, or the direction of the unit vector  $n = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$  and the component NP perpendicular to ON (parallel to the  $\pi$  plane, the deviatoric plane  $T_1+T_2+T_3 = 0$ ). Thus, the length of the component ON or hydrostatic-stress component  $\xi$  is

$$\xi = |ON| = ON \cdot n = (T_1+T_2+T_3)/\sqrt{3} = I_1/\sqrt{3} \quad (6.14.a)$$

and the length of NP or deviatoric-stress component, designated as  $\rho$ ,

$$\rho = |\text{NP}| = \sqrt{2J_2}. \quad (6.14.b)$$

Figure 6.1.b is a projection of OP, designated as  $\text{OP}'$ , and axes  $T_1$ ,  $T_2$  and  $T_3$ , designated as  $T_1'$ ,  $T_2'$  and  $T_3'$  on the  $\pi$  plane, where the angle of similarity  $\theta$  is an angle from the  $T_1'$ -axis to  $\text{OP}'$ , which can be shown to be defined by

$$\cos 3\theta = \frac{3/3 J_3}{2 J_2^{3/2}}. \quad (6.14.c)$$

The quantities  $\xi$ ,  $\rho$  and  $\theta$  can be considered three independent invariants of  $T$ , and the hydrostatic axis,  $T_1'$ -axis and  $\theta$  form a cylindrical coordinate system. For an isotropic material, the trace of its yield surface in the plane parallel to the  $\pi$ -plane at distance  $\xi$  from the origin is symmetric with respect to any one of axes  $T_1'$  and to any one of the perpendiculars to these axes through N.

The Willam-Warnke surface is defined in  $\sigma_n$  ( $= \xi/\sqrt{3}$ ),  $\rho$ ,  $\theta$  space by an equation of the form  $\rho - r(\sigma_n, \theta) = 0$ . The function  $r(\sigma_n, \theta)$  is chosen to fit the traces of this surface in two meridian planes,  $\theta = 0$  and  $\theta = \pi/3$ , and in a plane parallel to the deviatoric plane to experimental data. In the two meridian planes, the planes containing the hydrostatic axis with  $\theta = 0$  and  $\pi/3$  respectively, the Willam-Warnke model has curved tensile and compressive meridians, which are expressed by a quadratic parabola (Figure 6.2.a). Because of presuming concrete isotropic,

any deviatoric cross-section of the model is threefold symmetric as well as symmetric with respect to any deviatoric-stress axis. It looks like a bulged triangle with six parts, each of which is part of an ellipse (Figure 6.2.b). The two meridian equations have five parameters, which are coefficients in the formulas,

$$\left(\frac{\sigma_a}{f_c}\right) = a_0 + a_1\left(\frac{r_t}{f_c}\right) + a_2\left(\frac{r_t}{f_c}\right)^2 \quad \text{for } \theta = 0 \quad (6.15.a)$$

$$\left(\frac{\sigma_a}{f_c}\right) = b_0 + b_1\left(\frac{r_c}{f_c}\right) + b_2\left(\frac{r_c}{f_c}\right)^2 \quad \text{for } \theta = \pi/3 \quad (6.15.b)$$

where  $f_c$  is the static compressive strength;  $\sigma_a = I_1/3$  is the mean normal stress; and  $r_t$  and  $r_c$  are stress components perpendicular to the hydrostatic axis at the angles of similarity  $\theta = 0$  and  $\theta = \pi/3$ , respectively. In order to avoid confusion, character  $r$  is used to designate the deviatoric stress component of the point on the Willam-Warnke surface and  $\rho$  will be only used for the deviatoric stress component of the stress state of the material, though some authors use the same notation for both. Since the two meridians meet on the hydrostatic axis,  $a_0 = b_0$ . Therefore there are not six, but five parameters in the two quadratic functions.

At a mean normal stress  $\sigma_a$ , the deviatoric trace between  $\theta = 0$  and  $\theta = \pi/3$  is part of an ellipse with  $\theta = 0$  as one of its principal axes. The lengths of the two

ellipse semiaxes are determined in such a way that the ellipse must intersect the axes  $\theta = 0$  and  $\theta = \pi/3$  with a right angle at  $r = r_t$  and  $r = r_c$  respectively. After the Cartesian description of the ellipse is transformed into the polar coordinates  $(r, \theta)$  with the origin at O, which is not the center of the ellipse, the ellipse is expressed in the form of

$$r(\sigma_n, \theta) = \frac{s + t}{v} \quad (6.16.a)$$

$$\text{where } s = 2r_c(r_c^2 - r_t^2) \cos \theta \quad (6.16.b)$$

$$t = r_c(2r_t - r_c)u^{1/2} \quad (6.16.c)$$

$$u = 4(r_c^2 - r_t^2) \cos^2 \theta + 5r_t^2 - 4r_t r_c \quad (6.16.d)$$

$$v = 4(r_c^2 - r_t^2) \cos^2 \theta + (r_c - 2r_t)^2 \quad (6.16.e)$$

$$\theta = \frac{1}{3} \arccos\left(\frac{3/3 J_3}{2 J_2^{3/2}}\right) \quad (6.16.f)$$

The deviatoric trace is so defined that it is guaranteed to be continuous and smooth at  $\theta = 0$  and  $\pi/3$ , and also convex everywhere. According to Equation (6.16.a), the Willam-Warnke surface can be expressed as

$$G(\sigma_n, \sqrt{J_2}, \theta) = \sqrt{2J_2} - r(\sigma_n, \theta) = 0$$

$$\text{or } G(\sigma_n, \sqrt{J_2}, \theta) = \rho - r(\sigma_n, \theta) = 0 \quad (6.17)$$

where  $\rho = \sqrt{2J_2}$  is the deviatoric stress component of the current stress state of the material  $(\sigma_n, \sqrt{2J_2}, \theta)$ , and  $r(\sigma_n, \theta)$  is the radius of the fixed Willam-Warnke surface at  $\sigma_n$  and  $\theta$  in the cylindrical coordinate system. If  $\theta$  is calculated from (6.16.f) and only the principal value of the

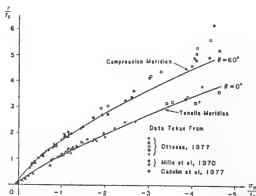
inverse cosine, which ranges from 0 to  $\pi$ , is taken,  $\theta$  ranges from 0 to  $\pi/3$ .

Because not enough static-test data of the high-strength concrete are available for curve fitting of the two meridians, the results from concrete of lower strength are used. The five-parameter values based on the tests by Ottosen, Mills et al. and Cedolin et al. (Figure 6.2) are  $a_0 = b_0 = 0.1025$ ,  $a_1 = -0.8403$ ,  $a_2 = -0.0910$ ,  $b_1 = -0.4507$  and  $b_2 = -0.1018$  (Chen, 1984). All available data of static compressive tests of the high-strength concrete with different loading paths varying  $\sigma_n$  and  $\rho$  with  $\theta$  kept zero (Bakhtar et al. 1985; Mould and Levine, 1987) are compared with the meridians determined by the above values of the five parameters (Figure 6.3). Gran's data (after Mould and Levine, 1987) coincide with the compressive meridian quite well. The parameters correlating the two nondimensional quantities  $\sigma_n/f_c$  and  $r/f_c$  seem independent of the static compressive strength  $f_c$  and other properties.

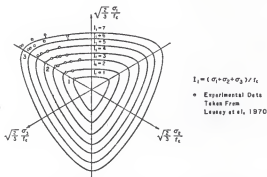
#### 6.2.4 Proposed Elastoplastic Cap Model for High-Strength Concrete

A rate-independent plastic model is proposed here. It consists of the Willam-Warnke surface as a perfect-yield surface fixed in the principal-stress space and a hardening cap-shaped surface assembled on the open end of the Willam-Warnke surface. The deviatoric trace of the cap is proposed





(a) Tensile and Compressive Meridians of the Willam-Warnke Surface



(b) Deviatoric Section of the Willam Warnke Surface

Note: Principal stresses are designated as  $\sigma_i$  in Figure 6.2 but as  $T_i$  in the context.

Figure 6.2 The Willam-Warnke Five-Parameter Failure Criterion (after Chen, 1985)

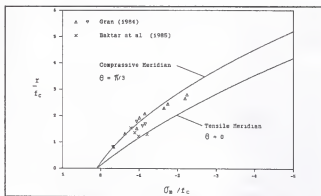
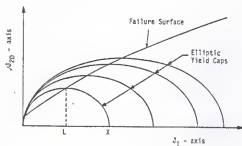


Figure 6.3 Tensile and Compressive Meridians of the Willam-Warnke Surface and High-Strength Concrete Test Data



Note:  $J_1$  and  $J_{2D}$  in the figure are the first stress invariant and the second deviatoric stress invariant respectively, which are designated as  $I_1$  and  $J_2$  in the context.

Figure 6.4 Meridians of the Failure Surface and Elliptic Yield Caps That Pass through the Origin (after Faruque, 1987)



to be given by the same expression as that of the Willam-Warnke surface, that is, expressed by Equations (6.16) and suitable reflections, but the tensile and compressive meridians are ellipses rather than quadratic parabolas. These elliptical cap meridians pass through the origin of the stress space and intersect at their highest points with the failure surface (which is the Willam-Warnke surface in the proposed model), as Faruque (1987) suggested, so that only one additional parameter is required, the length of the ellipse axis section along the hydrostatic-stress axis, shown as  $X$  in Figure 6.4. The hardening index  $X$  determines both the current size and location of the cap. It increases monotonically with the maximum compressive plastic volume strain the material has encountered in history. Faruque (1987) and some others use the evolution law for concrete as follows.

$$X = \frac{1}{D} \ln \left( 1 - \frac{\epsilon_v^p}{W} \right) + Z \quad (6.18)$$

where  $\epsilon_v^p$  is the maximum volumetric plastic strain having ever appeared at a point of the material;  $D$ ,  $W$  and  $Z$  are material parameters; and  $Z = 0$  when the material has no initial elastic range. This law was originally proposed by Dimaggio and Sandler (1971). This function is concave upward in the  $X$ - $\epsilon_v^p$  diagram, which may reflect the hardening relationship for soil and some kinds of concrete. However, the curve for the high-strength concrete tested in this

research program is concave downward, and therefore Dimaggio's form is not appropriate. The evolution law in the model proposed here is also a nondimensional equation, expressed as

$$\epsilon_v^p = d_1 \left( \frac{X}{f_c} - Z \right) + d_2 \left( \frac{X}{f_c} - Z \right)^2 + d_3 \left( \frac{X}{f_c} - Z \right)^3 \quad (6.19)$$

where  $d_1$ ,  $d_2$ ,  $d_3$ , and  $Z$  are all material parameters;  $X$  is the length of the axis section along the hydrostatic-stress axis of the current cap, and  $Z$  is the ratio of  $X$  of the initial cap over  $f_c$ .

In conclusion, the cap surface is proposed so that it can be expressed as

$$F(\sigma_n, \sqrt{J_2}, \theta, X) = \sqrt{2J_2} - r(\sigma_n, \theta, X) = 0$$

$$\text{or } F(\sigma_n, \sqrt{J_2}, \theta, X) = \rho - r(\sigma_n, \theta, X) = 0 \quad (6.20)$$

where  $r$  is also calculated from Equations (6.16) but  $r_t$  and  $r_c$  in (6.16) are not determined by Equations (6.15), but by ellipse equations as follows.

$$\left( \frac{r_t}{Y_t} \right)^2 + \left( \frac{\sigma_n}{X} \right)^2 = 1 \quad (6.21.a)$$

$$\left( \frac{r_c}{Y_c} \right)^2 + \left( \frac{\sigma_n}{X} \right)^2 = 1 \quad (6.21.b)$$

where  $X$  is determined by Equation (6.19), and  $Y_t$  and  $Y_c$  are solutions to Equations (6.15.a) and (6.15.b) respectively when  $\sigma_n = X/2$ .

The points calculated from two Terra Tek static-test curves with different loading paths (Figures 6.5 and 6.6)

and their least-square regression curve are marked in Figure 6.7 to show agreement of the experimental data with the proposed cap. One loading path (Test 19) was a proportional loading where  $T_1/T_2(=T_3) = 4/1$ . The other (Test 27) was a longitudinal compressive loading in confining pressure of 5 ksi. Coefficients for Equation 6.18 were solved as follows:  $d_1 = 2.719 \times 10^{-4}$ ,  $d_2 = -5.650 \times 10^{-4}$ ,  $d_3 = 1.360 \times 10^{-3}$ , and  $Z = 0.400$ .

#### 6.2.5 Plastic Strain Increments in the Proposed Cap Model

The associative flow rule is taken in the proposed cap model so that the plastic strain increments in loading are as follows.

$$dc_{ij}^p = \lambda \frac{\partial f(T)}{\partial T_{ij}} \quad (6.22)$$

where the proportionality factor  $\lambda$  is determined by the consistency equation or Equation (6.13), and  $f(T) = 0$  is the loading surface in the nine-dimensional stress space.

Function  $f(T)$  is identical to  $F(\sigma_m, \sqrt{2J_2}, \theta, X)$ , which was defined in the preceding section in the three-dimensional stress space when the current cap axis section along the hydrostatic-stress axis  $X$  has been determined, if the arguments in  $F$ ,  $\sigma_m$ ,  $\sqrt{2J_2}$  and  $\theta$ , are expressed in terms of  $T_{ij}$ . Therefore, among Equations (6.22),

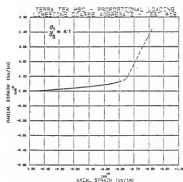


Figure 6.5 A Static Test of High-Strength Concrete by Terra Tek - Test 19 (after Bakhtar et al., 1985)

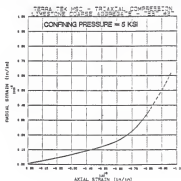


Figure 6.6 A Static Test of High-Strength Concrete by Terra Tek - Test 27 (after Bakhtar et al., 1985)

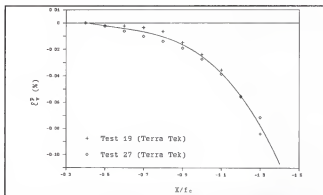


Figure 6.7 Inelastic Volume Strain Dependence of Parameter  $X$  of the Cap

$$\begin{aligned}\frac{\partial f}{\partial T_{1j}} &= \frac{\partial F}{\partial I_1} \frac{\partial I_1}{\partial T_{1j}} + \frac{\partial F}{\partial J_2} \frac{\partial J_2}{\partial T_{1j}} + \frac{\partial F}{\partial J_3} \frac{\partial J_3}{\partial T_{1j}} \\ &= C_1 \frac{\partial I_1}{\partial T_{1j}} + C_2 \frac{\partial J_2}{\partial T_{1j}} + C_3 \frac{\partial J_3}{\partial T_{1j}}\end{aligned}\quad (6.23.a)$$

$$C_1 = \frac{\partial F}{\partial I_1} \quad (6.23.b)$$

$$C_2 = \frac{\partial F}{\partial J_2} \quad (6.23.c)$$

$$C_3 = \frac{\partial F}{\partial J_3} \quad (6.23.d)$$

$$\frac{\partial I_1}{\partial T_{1j}} = \left\{ \begin{array}{ll} 1 & (\text{for } i = j) \\ 0 & (\text{for } i \neq j) \end{array} \right\} = \delta_{1j} \quad (6.23.e)$$

$$\frac{\partial J_2}{\partial T_{1j}} = s_{1j} \quad (6.23.f)$$

$$\frac{\partial J_3}{\partial T_{1j}} = s_{1k}s_{kj} - \frac{2}{3}J_2\delta_{1j} \quad (6.23.g)$$

After tedious manipulation the coefficients  $C_1$ ,  $C_2$  and  $C_3$  were obtained. The results of their expressions are summarized in the following.

$$\begin{aligned}C_1 &= -\frac{r'_c}{v} [2(3r_c^2 - r^2)\cos\theta - 2(r_c - r)u^{1/2} + 2r_c(2r_t - r_c)u^{1/2} \\ &\quad (2r_c\cos^2\theta - r) - 8rr_c\cos^2\theta - 2r(r_c - 2r_t)] \\ &\quad -\frac{r'_t}{v} [-4r_c r_t \cos\theta + 8r_t \cos^2\theta + 4r(r_c - 2r_t) + 2r_c u^{1/2} \\ &\quad - r_c(2r_t - r_c)u^{-1/2}(4r_t \cos\theta - 5r_t + 2r_c)]\end{aligned}\quad (6.24.a)$$

$$C_2 = \frac{1}{r} + \frac{3/3 (r_c^2 - r_t^2) J_3}{2v(4\cos^2\theta - 1)J_2^{5/2}} \{4r\cos\theta - r_c[1+2u^{-1/2}(2r_t - r_c)\cos\theta]\} \quad (6.24.b)$$

$$C_3 = \frac{3/3 (r_c^2 - r_t^2)}{v(4\cos^2\theta - 1)J_2^{3/2}} \{4r\cos\theta - r_c[1+2u^{-1/2}(2r_t - r_c)\cos\theta]\} \quad (6.24.c)$$

where

$$r'_t = \frac{dr_t}{dI_1} = - \frac{2\sigma_m - X}{3/[X^2 - (2\sigma_m - X)^2]} Y_t, \quad (6.25.a)$$

$$\text{and } r'_c = \frac{dr_c}{dI_1} = - \frac{2\sigma_m - X}{3/[X^2 - (2\sigma_m - X)^2]} Y_c. \quad (6.25.b)$$

Since the functions for  $C_2$  and  $C_3$  become the indeterminate form of  $0/0$  as  $\theta$  approaches  $\pi/3$ , it is necessary to work out their limits. The results are

$$C_2 = \frac{v}{ur_c} \quad \text{for } \theta = \pi/3, \quad (6.26.a)$$

and

$$C_3 = \frac{3/6 (r_c^2 - r_t^2)}{ur_c^2} \quad \text{for } \theta = \pi/3. \quad (6.26.b)$$

From Equation (6.21),

$$dF = C_1 dI_1 + C_2 dJ_2 + C_3 dJ_3 + \frac{\partial F}{\partial X} \frac{dX}{d\epsilon_{kk}^p} d\epsilon_{kk}^p, \quad (6.27)$$

but

$$d\epsilon_{kk}^p = 3\lambda \frac{\partial F}{\partial I_1}, \quad (6.28)$$

therefore,

$$\lambda = - \frac{C_1 dI_1 + C_2 dJ_2 + C_3 dJ_3}{3C_1 \frac{\partial F}{\partial X} \frac{dX}{d\epsilon_{kk}^p}} \quad (6.29)$$

where

$$\frac{d\epsilon_{kk}^p}{dX} = d_1 + 2d_2(X-Z) + 3d_3(X-Z)^2 \quad (6.30)$$

from Equation (6.19), and the partial derivative  $\partial F/\partial X$  is derived as follows. For convenience,  $r_t$  and  $r_c$  are written as

$$r_t = Y_t q \quad (6.31.a)$$

$$r_c = Y_c q \quad (6.31.b)$$

where

$$q = \frac{2}{X} \sqrt{[(X/2)^2 - (\sigma_m - X/2)^2]} \quad (6.31.c)$$

Thus

$$\frac{\partial r_t}{\partial X} = Y_t \frac{\partial q}{\partial X} + \frac{\partial Y_t}{\partial X} q, \quad (6.32.a)$$

and

$$\frac{\partial r_c}{\partial X} = Y_c \frac{\partial q}{\partial X} + \frac{\partial Y_c}{\partial X} q, \quad (6.32.b)$$

where

$$\frac{\partial q}{\partial X} = \frac{2\sigma_m}{X^2 q} \left( \frac{2\sigma_m}{X} - 1 \right), \quad (6.31.c)$$

$$\frac{\partial Y_t}{\partial X} = - \frac{1}{\sqrt{[a_1^2 - 4a_2(a_0 - X/2f_c)^2]}} \quad (6.32.d)$$

and

$$\frac{\partial Y_c}{\partial X} = - \frac{1}{\sqrt{[b_1^2 - 4b_2(b_0 - X/2f_c)^2]}} \quad (6.32.e)$$



Furthermore,

$$\partial s / \partial r_t = -4Y_t Y_c q^2 \cos \theta ,$$

$$\partial s / \partial r_c = 2(3Y_c^2 - Y_t^2) q^2 \cos \theta ,$$

$$\partial t / \partial r_t = q^2 [2Y_c u^{1/2} - Y_c (2Y_t - Y_c) (4Y_t \cos^2 \theta - 5Y_t + 2Y_c) u^{-1/2}] ,$$

$$\partial t / \partial r_c = q^2 [-2(Y_c - Y_t) u^{1/2} + 2Y_c (2Y_t - Y_c) (2Y_c \cos^2 \theta - Y_t) u^{-1/2}] ,$$

$$\partial v / \partial r_t = q [-8Y_t \cos^2 \theta - 4(Y_c - 2Y_t)] ,$$

$$\text{and } \partial v / \partial r_c = q [8Y_c \cos^2 \theta + 2(Y_c - 2Y_t)] . \quad (6.33)$$

Then the following partial derivatives are obtained with the chain rule.

$$\frac{\partial s}{\partial X} = \frac{\partial s}{\partial r_t} \frac{\partial r_t}{\partial X} + \frac{\partial s}{\partial r_c} \frac{\partial r_c}{\partial X} , \quad (6.34.a)$$

$$\frac{\partial t}{\partial X} = \frac{\partial t}{\partial r_t} \frac{\partial r_t}{\partial X} + \frac{\partial t}{\partial r_c} \frac{\partial r_c}{\partial X} , \quad (6.34.b)$$

and

$$\frac{\partial v}{\partial X} = \frac{\partial v}{\partial r_t} \frac{\partial r_t}{\partial X} + \frac{\partial v}{\partial r_c} \frac{\partial r_c}{\partial X} . \quad (6.34.c)$$

Finally,  $\partial F / \partial X$  is obtained as

$$\begin{aligned} \frac{\partial F}{\partial X} &= - \frac{\partial}{\partial X} \left( \frac{s + t}{v} \right) \\ &= \frac{1}{v} \left( - \frac{\partial s}{\partial X} - \frac{\partial t}{\partial X} + r \frac{\partial v}{\partial X} \right) . \end{aligned} \quad (6.35)$$

### 6.3 An Elastoviscoplastic Model for High-Strength Concrete

#### 6.3.1 Proposed Elastoviscoplasticity Model

The ultimate purpose of the study on properties of concrete under high strain rates is to construct its constitutive equations which are likely to be applied to computation codes so that engineers can use the research results to predict the response of concrete structures to dynamic loading. A continuum elastoviscoplastic model is proposed in this section.

This model is an extension of Malvern's overstress model to the Willam-Warnke perfect-yield surface. The overstress is defined as  $\sqrt{3/2}$  times the difference of the deviatoric stress in principal-stress space,  $\sqrt{2J_2}$ , and the radius of the Willam-Warnke surface  $r(\sigma_n, \theta)$  at the same mean normal stress  $\sigma_n$  and angle of similarity  $\theta$ .

$$\{\sigma\} = \sqrt{3/2} [\sqrt{2J_2} - r(\sigma_n, \theta)] = [\sqrt{3/2}]G \quad (6.36)$$

where  $\{\sigma\}$  is overstress. The factor  $\sqrt{3/2}$  makes  $\{\sigma\}$  be  $[T_{11} - \sqrt{3/2} r_c]$  for the uniaxial stress case to provide convenience in using experimental data from uniaxial tests. When  $\{\sigma\}$  is zero, the stress state lies on the Willam-Warnke surface. A modified associative flow rule is assumed that the model is expressed as

$$\dot{\epsilon}_{ij}^{vp} = \gamma \langle \phi(G) \rangle \frac{\partial g}{\partial T_{ij}} \left( \frac{\partial g}{\partial T_{km}} \frac{\partial g}{\partial T_{km}} \right)^{-1/2} \quad (6.37)$$

where

$$\langle \phi(G) \rangle = \begin{cases} 0 & \text{for } G \leq 0 \\ \phi(G) & \text{for } G > 0 \end{cases}, \quad (6.38)$$

and  $G = 0$  is the Willam-Warnke surface defined by Equation (6.17). For  $G > 0$  the current stress point is outside the Willam-Warnke surface. In Equation (6.37) the derivatives  $\partial G / \partial T_{ij}$  are, however evaluated at the point on the Willam-Warnke surface which has the same values of  $\sigma_m$  and  $\theta$  as the current stress point. The viscosity parameter  $\gamma$  and the form of the viscosity function  $\phi(F)$  are determined by experimental observation, which will be given in the next subsection. For the partial derivatives in Equations (6.37), all the formulas of (6.23), (6.24) and (6.26) are valid so long as every  $f$  and  $F$  in these equations is replaced by  $g$  and  $G$  respectively, where  $g(T) = 0$  is the Willam-Warnke surface expressed in the nine-dimensional  $T_{ij}$  space, and  $G(\sigma_m, \rho, \theta) = 0$  is the surface expressed in the three-dimensional stress space. However,  $r'_t$  and  $r'_c$  in these equations should be calculated as follows.

$$r'_t = \frac{dr_t}{dI_1} = - \frac{1}{3[a_1 + 2a_2(r/r_c)]}, \quad (6.39.a)$$

and

$$r'_c = \frac{dr_c}{dI_1} = - \frac{1}{3[b_1 + 2b_2(r/r_c)]}. \quad (6.39.b)$$

Because of a shortage of data from tests under three-dimensional dynamic loading, the hardening cap is assumed strain-rate independent in the proposed model. In summary,

when  $G \leq 0$  and  $|\sigma_m| < X/2$ , or  $F \leq 0$  and  $|\sigma_m| \geq X/2$ , concrete is modeled as an isotropic linear elastic material; when  $F = 0$ ,  $dT_{ij}dF_{ij} > 0$  and  $|\sigma_m| > X/2$ , it is elastoplastic and plastic strain increments are given by Equation (6.22); and when  $G > 0$  and no matter how high  $\sigma_m$  is, concrete is modeled as elastoviscoplastic and increments in the inelastic strain part or viscoplastic strain follow Equation (6.37).

### 6.3.2 Determination of Viscosity Parameter and Function

Viscosity function  $\phi$  and viscosity parameter  $\gamma$  are determined by reducing the model to the uniaxial stress case.

In the uniaxial stress case (6.36) becomes

$$(\sigma) = T_{11} - \sqrt{(3/2)} r_c(T_{11}/3) \quad (6.40)$$

where  $r_c(T_{11}/3)$  is determined by (6.15.b) with  $\sigma_m = T_{11}/3$ . According to (6.37), overstress  $(\sigma)$ , not stress-tensor components, is related to the viscoplastic strain-rate tensor components, though, shown in Section 4.1, the longitudinal normal stress beyond yield simply follows a well-fitted semilogarithmic or power-law function in terms of current inelastic strain rate (Formulas 3.1 and 3.2) in the uniaxial stress case. Maximum overstresses for all the high-strength WES and SRI concrete specimens were calculated with (6.40), and the maximum overstress versus strain rate at the maximum overstress were fitted by regression to a

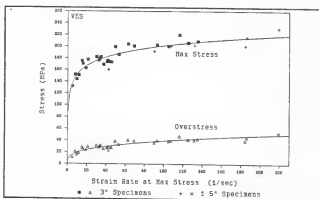


Figure 6.8 Inelastic Strain Rate Dependence of Overstress and Stress Beyond Yield for WES High-Strength Concrete Specimens with Stress in Units of MPa

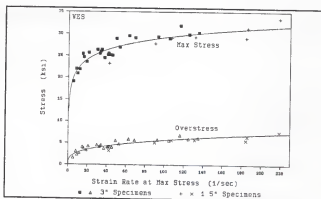


Figure 6.9 Inelastic Strain Rate Dependence of Overstress and Stress Beyond Yield for WES High-Strength Concrete Specimens with Stress in Units of ksi

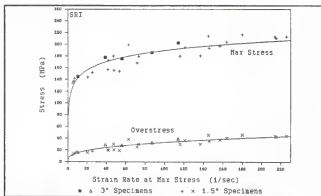


Figure 6.8 Inelastic Strain Rate Dependence of Overstress and Stress Beyond Yield for SRI High-Strength Concrete Specimens with Stress in Units of MPa

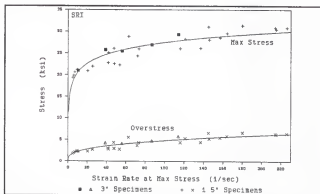


Figure 6.11 Inelastic Strain Rate Dependence of Overstress and Stress Beyond Yield for SRI High-Strength Concrete Specimens with Stress in Units of ksi

power-law function given in (6.41) (The maximum overstress took place at the maximum stress.)

$$\{\sigma\} = \kappa_o (\dot{\epsilon}^I / \dot{\epsilon}_o^I)^{\delta} \quad (6.41.a)$$

$$\text{or} \quad \dot{\epsilon}^I = \dot{\epsilon}_o^I [\{\sigma\} / \kappa_o]^{1/\delta} \quad (6.41.b)$$

where  $\dot{\epsilon}_o^I = -1/\text{sec}$  is the reference strain rate. The maximum overstress versus strain rate at maximum overstress points and their semilogarithmic regression curves are plotted in Figures 6.8 to 6.11 respectively for WES and SRI specimens. In the four figures compressive strain rate is considered positive, while it is considered negative in all the equations in this chapter. To compare with the overstress versus strain rate relation, also shown in Figures 6.8 to 6.11 are the maximum stress versus strain rate at maximum stress points and their regression curves, which have been all shown in Figures 3.29 to 3.32. Coefficients solved for WES and SRI respectively are listed in Table 6.1. As regression for maximum stress versus strain-rate at maximum stress statistically fits the stress development with inelastic-strain-rate in a single specimen as seen in Chapter 4, Equation (6.41) fits the overstress versus inelastic-strain-rate relation for a single specimen too.

By comparing Equation (6.41.b) with Equation (6.37),  $\Phi(G)$  and  $\gamma$  are obtained as

$$\Phi(G) = [\sqrt{(3/2)G/\kappa_o}]^{1/\delta} \quad (6.42)$$

$$\text{and} \quad \gamma = |\dot{\epsilon}_o^I| = 1/\text{sec} \quad (6.43)$$

for Equation (6.37).

Table 6.1 Viscosity Coefficients for WES  
and SRI High-Strength Concretes

|            | WES      | SRI      |
|------------|----------|----------|
| $\kappa_0$ | 1.35 ksi | 1.02 ksi |
| $\delta$   | 0.311    | 0.336    |

### 6.3.3 Discussion

The elastoviscoplastic model proposed is based on the Willam-Warnke surface. The Willam-Warnke model involves physical ideas that octahedral shear stress causes static fracture and that the static shear strength of concrete increases with the magnitude of hydrostatic or octahedral stress  $\sigma_m$  and the orientation  $\theta$ . Overstress is defined as the current octahedral stress  $\tau_o$  ( $=\rho/\sqrt{3}$ ) minus the octahedral shear strength  $r/\sqrt{3}$  determined by the other two stress invariants  $\sigma_m$  and  $\theta$ . According to the proposed model, the inelastic strain rate tensor components, given by Equation (6.37), depend upon the overstress

$[\sqrt{(3/2)}]G = \tau_o - r/\sqrt{3}$ . Components of inelastic strain rate tensor are proportional to the direction cosines of the outward normal to the Willam-Warnke surface expressed in the nine-dimensional stress space at a point on the surface. This point on the Willam-Warnke surface is such a point that two stress invariants  $\sigma_m$  and  $\theta$  of the stress state this point stands for are identical to those of the current



stress tensor of material.

Viscosity function and parameter are obtained from data of uniaxial dynamic tests. In these tests, the loading path is a straight line in the meridian plane  $\theta = \pi/3$  with parameter equations  $\sigma_m = T_{11}/3$  and  $\rho = \sqrt{(2/3)}T_{11}$ , and has an intersection with the compressive meridian at point  $(\sigma_m = f_c/3, \rho = \sqrt{(2/3)}f_c)$ . Along the loading path but outside the meridian or  $\sigma_m > f_c$ , the octahedral shear stress of the stress state  $\tau_o = \sqrt{(2J_2/3)} = (\sqrt{2/3})T_{11}$  is greater than  $\tau_o(\sigma_m)/\sqrt{3}$ , the octahedral shear stress component of the point on the compressive meridian at the same  $\sigma_m$ . Analysis in Section 3.4 shows no significant radial inertia pressure existed in the tests. However, the longitudinal stress in the specimen, which was up to twice the static strength in some tests, raised the hydrostatic stress, and the hydrostatic stress would enhance shear strength of concrete according to the Willam-Warnke model. In order to show pure strain rate effects, Equation (6.41) was obtained to get rid of the octahedral shear stress enhancement caused by hydrostatic stress.

Failure criterion and postfailure behavior are not included in the proposed model. Probably a continuum model may not fit the postfailure or strain softening stage because considerable cracks have existed. The hardening cap in the proposed model is assumed independent of strain rate only because of a shortage of data. Dynamic tests with

different hydrostatic stress are needed, and stress states inside the Willam-Warnke surface should be well studied in future tests.

The proposed model has been applied to a time-varying finite element program. The application will be shown in next chapter.

CHAPTER 7  
IMPLEMENTATION OF THE ELASTOVISCOPLASTIC MODEL  
IN A TIME-VARYING FINITE ELEMENT PROGRAM

7.1 A Finite Element Program in Dynamic Analysis

7.1.1 Introduction

The finite element method has rapidly become a very popular technique for the computer solution of complex problems in engineering during the past two decades. Meanwhile, it has been realized that the advances and sophistication in the solution technique have far exceeded our knowledge of the behavior of materials defined by constitutive laws. Since the present work primarily aims at the behavior of concrete, neither history of the finite element method nor details of general procedures of this technique for stress analysis are reviewed; these are available in various textbooks. Because this chapter is to implement the elastoviscoplastic model proposed in Chapter 6 to an existing time-varying finite element program, only theoretical formulas relevant to the finite element program will be presented. The original program, named DYNPAK and written in FORTRAN, is based on the explicit time integration scheme. The whole program is listed in Chapter

10 of the book *Finite Elements in Plasticity* by D. R. J. Owen and E. Hinton (1980). In this work, the main program of DYNPAK was improved, and some additional subprograms were written and added in the program in order to include the proposed elastoviscoplastic model. The major differences between DYNPAK and the modified program, named UFCONC, are (1) that DYNPAK is applicable to elastoviscoplastic materials based on Tresca, von Mises, Mohr-Coulomb and Prager-Drucker yield surfaces while UFCONC is applicable to not only these materials but also materials that follow the proposed model; (2) that loads in DYNPAK can only be a Heaviside or harmonic function, which is generated by the program itself, but in UFCONC loads can be input from external data files so that any recorded impulses, and any preprepared numerical or analytical functions may be taken as loads; and (3) that in DYNPAK only a single form of function is admissible for all loads, even though they distribute at different locations, while UFCONC admits loads in different forms at different boundaries.

Like DYNPAK, UFCONC performs large displacement or elastoviscoplastic or elastoplastic, transient dynamic analysis of plane stress/strain or axisymmetric problems respectively. Isoparametric 4, 8 and 9-noded quadrilateral elements are included in the program. A special mass lumping procedure has been adopted and separate Gauss-

Legendre rules may be adopted in the evaluation of the stiffness and the lumped mass matrices.

### 7.1.2. Dynamic Equilibrium Equations

The Principle of Virtual Displacements is used to write the dynamic equilibrium equations at time step  $t_n$  as follows.

$$\int_{\Omega} [\delta \epsilon_n]^T \sigma_n d\Omega - \int_{\Omega} [\delta u_n]^T [b_n - \rho_n \ddot{u}_n - c_n \dot{u}_n] d\Omega - \int_{\Gamma} [\delta u_n]^T t_n d\Gamma = 0 \quad (7.1)$$

where  $\delta u_n$  is the vector of virtual displacements,  $\delta \epsilon_n$  is the vector of associated virtual strains,  $b_n$  is the vector of applied body forces,  $t_n$  is the vector of surface tractions,  $\sigma_n$  is the vector of stresses,  $\rho_n$  is the mass density,  $c_n$  is the damping parameter and a dot refers to differentiation with respect to time. Here the word vector is used to mean column matrix as is common in the computational literature. The domain of interest  $\Omega$  has two boundaries:  $\Gamma$  on which boundary tractions  $t_n$  are specified and  $\Lambda$  on which displacements  $u_n$  are specified.

For a finite element representation, the displacements and also their virtual counterparts are given by the relationships

$$u_n = N_i [d_i]_n, \quad \delta u_n = N_i [\delta d_i]_n \quad (7.2)$$

$$\epsilon_n = B_i [d_i]_n, \quad \delta \epsilon_n = B_i [\delta d_i]_n \quad (7.3)$$

where at time  $t_n$  for node  $i$ ,  $[d_i]_n$  is the vector of nodal displacements,  $[\delta d_i]_n$  is the vector of virtual nodal variables,  $N_i = N_i I$  is the matrix of global shape functions

and  $B_i$  is the global strain-displacement matrix. The total number of nodes is  $m$ , and  $i=1,2,\dots,m$  in (7.2) and (7.3).

Because Equation (7.1) is true for any set of virtual displacements  $[\delta d_i]_n$ , the dynamic equilibrium equations become

$$M_n \ddot{d}_n + C_n \dot{d}_n + K_n d_n = f_n \quad (\text{no sum on } n) \quad (7.4)$$

where  $[M_{ij}]_n = \int_0 [N_i]^T \rho_n [N_j] d\Omega$  is a submatrix of the mass matrix  $M_n$ ,  $[C_{ij}]_n = \int_0 [N_i]^T c_n [N_j] d\Omega$  is a submatrix of the damping matrix  $C_n$ ,  $[K_{ij}]_n = \int_0 [B_i]^T D [B_j] d\Omega$  is a submatrix of the stiffness matrix  $K_n$ , and  $[f_i]_n = \int_0 [N_i]^T t_n d\Omega + \int_r [N_i]^T t_n dr$  is the total consistent forces for the applied body forces and traction boundary forces. Of course, the subscripts  $i$  and  $j$  in the above formulas can be left off if it is assumed that  $N=[N_1 \ N_2 \dots N_n]$  and  $B=[B_1 \ B_2 \dots B_n]$ . Alternatively,  $M_n$ ,  $C_n$ ,  $K_n$  and  $f_n$  can be assembled by the usual rule from the element submatrices given by

$$[M^*]_n = \int_{\Omega^*} [N^*]^T \rho_n [N^*] d\Omega^* \quad (7.5)$$

$$[C^*]_n = \int_{\Omega^*} [N^*]^T c_n [N^*] d\Omega^* \quad (7.6)$$

$$[K^*]_n = \int_{\Omega^*} [B^*]^T [D] [B^*] d\Omega^* \quad (7.7)$$

$$[f^*]_n = \int_{\Omega^*} [N^*]^T t_n d\Omega^* + \int_{r^*} [N^*]^T t_n dr^* \quad (7.8)$$

where  $i,j=1,2,\dots,r$  ( $r$  is the number of nodes in an element),

$D$  is the stress-strain matrix of the material,  $N^* =$

$[N_1 \ N_2 \dots N_r]$  is the matrix of local shape functions ( $N_i = N_i I$ ),

and  $B^* = [B_1 \ B_2 \dots B_r]$  is the local strain-displacement matrix.

When the strains are linear,  $B_i^*$  is independent of time and

therefore the element stiffness  $K_n^*$  and the global stiffness

$K_n$  are all independent of the time step  $n$ . For simplicity, the mass and damping matrices are often assumed not varying with time too.

Neglecting damping forces may cause severe errors or even make a solution impossible. It is customary to assume the artificial damping matrix  $C$  proportional to the mass and stiffness matrix

$$C = \alpha M + \beta K \quad (7.9)$$

in linearly-elastic analysis. With the elastoviscoplastic model, artificial damping is not required.

### 7.1.3 Implementation of the Elastoviscoplastic Model

The elastoviscoplastic model of overstress type, which is proposed in Chapter 6, is in the form of

$$\dot{\epsilon}_n = \dot{\epsilon}_n^e + \dot{\epsilon}_n^{vp} \quad (7.10)$$

$$[\dot{\epsilon}_i^{vp}]_n = \gamma \langle \Phi_n(G) \rangle \frac{\partial G}{\partial (\sigma_i)_n} \left( \frac{\partial G}{\partial (\sigma_k)_n} \frac{\partial G}{\partial (\sigma_k)_n} \right)^{-1/2} \quad (7.11)$$

(no sum on  $n$ )

where

$$\langle \Phi_n(G) \rangle = \begin{cases} 0 & \text{for } G \leq 0 \\ \Phi_n(G) & \text{for } G > 0 \end{cases}, \quad (7.12)$$

and  $G(\sigma_i)=0$  is the Willam-Warnke surface. The subscript  $n$  stands for variable values at time step  $n$ , and  $\sigma_i$  and  $\dot{\epsilon}_i^{vp}$  are respectively components of the stress vector and viscoplastic strain-rate vector. In the plane stress and plane strain cases, the stress and strain vectors are defined as

$$\sigma = [\sigma_x, \sigma_y, \tau_{xy}]^T \quad (7.13.a)$$

$$\text{and} \quad \epsilon = [\epsilon_x, \epsilon_y, \gamma_{xy}]^T, \quad (7.13.b)$$

while in the axisymmetric case, they are defined as

$$\sigma = [\sigma_r, \sigma_\theta, \sigma_z, \tau_{rz}]^T \quad (7.14.a)$$

$$\text{and} \quad \epsilon = [\epsilon_r, \epsilon_\theta, \epsilon_z, \gamma_{rz}]^T. \quad (7.14.b)$$

The stress vector is related to the elastic component of the strain vector by a linearly-elastic constitutive matrix D as

$$\sigma_n = D\epsilon_n^e \quad (7.15)$$

where

$$D = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (7.16)$$

for the plane stress case,

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2 \end{bmatrix} \quad (7.17)$$

for the plane strain case, and

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 & 0 \\ \nu & 1-\nu & \nu & 0 \\ 0 & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix} \quad (7.18)$$

for the axisymmetric case, and E and  $\nu$  are Young's modulus and Poisson's ratio respectively. From (7.10) and (7.15),

$$\sigma_n = D(\epsilon_n - \epsilon_n^{vp}). \quad (7.19)$$

Stresses at time step n are obtained from solved strains and viscoplastic strains at time step n by (7.19) in programs DYNPAK and UFCONC. With (7.19), the second integral on the left side of (7.1) yields not only elastic forces  $K_n u_n$  appearing in (7.4), but also nonlinear damping forces, which



are apparently dependent on the velocity. As a result, it is not necessary to tinker with an artificial damping matrix when the elastoviscoplastic model is used. Equation (7.4) is rewritten as

$$M_n \ddot{d}_n + p_n = f_n \quad (\text{no sum on } n) \quad (7.20)$$

where  $p_n = \int_{\Omega} B^T D(\epsilon_n - \epsilon_n^{vp}) d\Omega$  is the global vector of initial resisting forces and it can also be assembled from element representation  $p_n^e$ . For viscoplastic strains, the simplest, Euler, integration scheme is employed in DYNPAK and UFGCONC

$$\epsilon_{n+1}^{vp} = \epsilon_n^{vp} + \dot{\epsilon}_n^{vp} \Delta t \quad (7.21)$$

where  $\Delta t$  is the time step. In the proposed model, the hardening cap is assumed independent of strain rate, and plastic strain increments when the current stress is on the cap are determined by the associative flow rule and the consistency condition. Equation (7.20) is used in programming with  $\epsilon_n^{vp}$  replaced by  $\epsilon_n^p$  in the expression for  $p_n^e$ .

#### 7.1.4 The Central Difference Method

A central difference approximation is adopted in DYNPAK and UFGCONC so that the acceleration can be written as

$$\ddot{d} = \frac{1}{(\Delta t)^2} (d_{n+1} - 2d_n + d_{n-1}) \quad (7.22)$$

and the velocities are written as

$$\dot{d} = \frac{1}{2\Delta t} (d_{n+1} - d_{n-1}) \quad (7.23)$$

where  $\Delta t$  is the time step or interval. By substituting (7.22) and (7.23) in (7.4) or (7.20), an expression for  $d_{n+1}$  in terms of  $f_n$ ,  $d_n$  and  $d_{n-1}$  is obtained.

For the model proposed, Equation (7.20) is employed. If the mass matrix  $M$  is diagonal, the solution to (7.20) for plane stress and plane strain applications becomes

$$(d_{ui})_{n+1} = (m_{ii})^{-1}[(\Delta t)^2(-(p_{ui})_n + (f_{ui})_n) + 2m_{ii}(d_{ui})_n - m_{ii}(d_{ui})_{n-1}] \quad (7.24)$$

and

$$(d_{vi})_{n+1} = (m_{ii})^{-1}[(\Delta t)^2(-(p_{vi})_n + (f_{vi})_n) + 2m_{ii}(d_{vi})_n - m_{ii}(d_{vi})_{n-1}] \quad (7.25)$$

where the subscripts  $u$  and  $v$  represent the  $x$  and  $y$  directions, the subscript  $i$  represents the node number, and  $m_{ii}$  are the diagonal terms of the diagonal mass matrix (no sum on  $i$ ). For axisymmetric problems replace  $v$  by  $w$ , which represents the  $z$  direction.

The governing dynamic equilibrium equations (7.24) and (7.25) at time step  $n+1$  in the central difference method involve information at the two previous time steps  $n$  and  $n-1$ . A starting algorithm is therefore necessary to solve for  $d_1$ , and the value  $d_1$  may be obtained from the initial conditions. From (7.22), the initial velocity condition gives

$$d_1 = -2(\Delta t)v_0 + d_0 \quad (7.26)$$

where  $v_0$  is designated for the node initial velocity at the starting time  $t=0$ . By substituting (7.26) in (7.24),  $d_1$  is obtained

$$(d_{ul})_1 = (\Delta t)^2 (2m_{11})^{-1} \{ -(p_{ul})_0 + (f_{ul})_0 \} + (d_{ul})_0. \quad (7.27)$$

The diagonal (or lumped) mass matrix is necessary for the above explicit algorithm. For lumped mass matrix, see (Zienkiewicz, 1977). The explicit, central difference time stepping scheme is a simple and powerful method of time integration, but the scheme is conditionally stable. Small time steps are required for accurate and stable solution. The stability and accuracy analysis for the direct integration is discussed in (Bathe, 1982). The implicit time integration method is unconditionally stable but requires large computer core storage and more operations per time step than the central difference scheme. A program, named MIXDYN, for a combined implicit-explicit scheme for linear and nonlinear transient dynamic analysis is included in Chapter 11 of *Finite Elements in Plasticity* (Owen and Hinton, 1980). The new proposed model has not yet been introduced into MIXDYN.

#### 7.1.5 Some Considerations in Programming

As a major modification of DYNPAK, UFGCONC includes a subprogram to calculate plastic strain increments based on the strain-hardening cap, and viscoplastic strain increments based on the overstress model. In the subprogram, an index

is determined from the current stress state at a Gauss point, and is used to decide if elastic, elastoplastic or elastoviscoplastic analysis should be performed at the current time step.

Figure 7.1 shows that five zones are specified with different indices in a plane containing the hydrostatic-stress axis in the principal-stress space. At every time step, after stress tensor components are determined at a Gauss point, the stress invariants and deviatoric stress invariants, including  $\sigma_m$ ,  $\rho$  ( $=\sqrt{2J_2}$ ), and  $\theta$  ( $=(\text{Arccos}[3\sqrt{3}J_3/2J_2^{3/2}])/3$ ) are calculated. When  $\sigma_m/f_c > a_0$  (See (6.15) for  $a_0$ ), the index = 0. If  $\sigma_m/f_c \geq a_0$ , then  $G(\sigma_m, \sqrt{J_2}, \theta) = \rho - r(\sigma_m, \theta)$  as in (6.17) is evaluated, where  $r(\sigma_m, \theta)$  is the radius of the fixed Willam-Warnke surface. If  $G > 0$ , the index = 3; if  $G \leq 0$  and  $\sigma_m > X/2$ , the index = 1; if  $G \leq 0$  and meanwhile  $\sigma_m/f_c < X/f_c$  (See (6.19) for  $X$ ), the index = 5. Otherwise the index is 2 or 4.  $F(\sigma_m, \sqrt{J_2}, \theta) = \rho - r(\sigma_m, \theta)$ , where  $r(\sigma_m, \theta)$  is the radius of the cap surface, is evaluated. When  $F > 0$ , the index = 4; when  $F > 0$ , the index = 2.

Zones 0, 1 and 2 are elastic zones. When the current stress-state index is 0, 1 or 2, the subprogram goes to elastic analysis. Actually, if the stress state enters into Zone 0, concrete may be fractured, but the subprogram ignores it in order to keep the computation continuing.

When the current stress-state index is 3, the elastoviscoplastic model is used to calculate the viscoplastic strain increments according to Equation (6.37). In Figure 7.1, point P represents a current stress point in zone 3 where  $G > 0$ . Point Q is an associated point on the Willam-Warnke surface. P and Q have the same values of  $\sigma_e$  and  $\theta$ , but while the radial coordinates of P and Q are  $\rho/f_c$  and  $r(\sigma_e, \theta)/f_c$ . It can be shown the nine stress deviatoric components  $T'_{ij}$  associated with Q are  $(r/\rho)$  times the stress deviator components associated with P. In the evaluation of Equation (6.37) the derivatives are at the  $T'_{ij}$  point corresponding to Q, while the multiplier  $\langle \phi(G) \rangle$  is evaluated at the  $T'_{ij}$  point corresponding to P.

When the index for the current time step is 4 or 5, plastic analysis should be performed. For a Gauss point that just enters Zones 4 or 5 from an elastic zone, the intersection of the line connecting the current and previous stress states in the  $T'_{ij}$  space with the cap surface is located by the *bisection* technique (Figure 7.2). Plastic strain increments are obtained by Equation (6.22) with the normal to the cap surface at the intersection, and the proportion constant  $\lambda$ , which is in terms of the stress-invariant increments from the intersection to the current stress state.

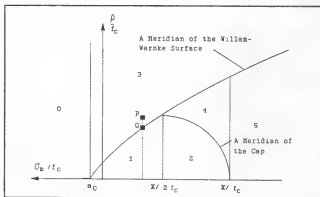


Figure 7.1 Indices for Deciding the Model to Be Used for Inelastic Strain Increments Computation

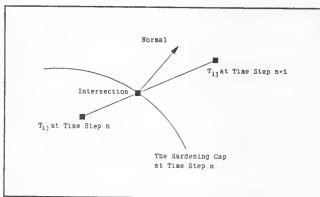


Figure 7.2 Stress State Crosses the Yield Surface in One Time Step Interval

## 7.2 Computation Examples - Analysis of Unconfined WES Specimen Tests

### 7.2.1 Introduction

The nonlinear transient dynamic analysis was made for the gaged 3-inch-long and 3-inch-diameter WES concrete specimen to show the reliability of the proposed model and UFCNC.

### 7.2.2 Geometry and Material Parameters

The 3-inch-long and 3-inch-diameter cylindrical WES concrete specimen is assumed subjected to evenly-distributed pressures on the two end surfaces, which are  $\sigma_1$  and  $\sigma_2$  acquired in the SHPB test. The lateral surface of the specimen is free from traction. Figure 7.3 shows the configuration of the specimen and finite element gridding for the axisymmetric stress case. The specimen is divided into 18 8-node and 0.5-inch-by-0.5-inch elements with three in the radial direction and six in the longitudinal direction. Specimens W24, W20, WT3 and W28 were analyzed. Pressures  $\sigma_1$  and  $\sigma_2$  from the four tests have been shown in Chapter 3, which are in a time interval of 0.5 microsecond. Linear interpolation was made between neighboring points in the  $\sigma_1$  and  $\sigma_2$  records to prepare a small enough time interval, 0.1 microsecond for W24 and W20 and 0.05 microsecond for WT3 and W28, to produce a stable and accurate solution. Shown in Table 7.1 are material parameters for WES specimens. Young's modulus and Poison's

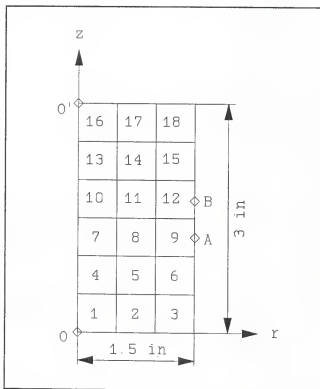


Figure 7.3 Finite Element Gridding for the 3-inch-long and 3-inch-diameter WES Specimen (Each element is filled by its element number.)



Table 7.1 Material Parameters for WES  
Concrete Input in Computation

|                                                    |                       |
|----------------------------------------------------|-----------------------|
| Young's Modulus $E$                                | See Table (3.8)       |
| Poisson's Ratio $\nu$                              | See Table (3.8)       |
| Static Compressive Strength $f_c$                  | 101 MPa<br>(14.6 ksi) |
| Constants for Willam-Warnke Surface <sup>(1)</sup> |                       |
| $a_0 = b_0$                                        | 0.1025                |
| $a_1$                                              | -0.8403               |
| $a_2$                                              | -0.0910               |
| $b_1$                                              | -0.4507               |
| $b_2$                                              | -0.1018               |
| Constants for the Hardening Cap <sup>(2)</sup>     |                       |
| $Z$                                                | 0.4000                |
| $D_1$                                              | 0.0002719             |
| $D_2$                                              | -0.0005650            |
| $D_3$                                              | 0.001360              |
| Viscosity Constants <sup>(3)</sup>                 |                       |
| $\dot{\epsilon}_0^{vp}$                            | -1.0/sec              |
| $\kappa_0$                                         | 1.35 ksi              |
| $\delta$                                           | 0.311                 |

<sup>(1)</sup> For Equation (6.15)

<sup>(2)</sup> For Equation (6.19)

<sup>(3)</sup> For Equations (6.37), (6.42) and (6.43)

ratio used in computation were from each individual test and all the other material parameters were summed up from a number of different tests.

Friction may exist on the interfaces between the specimen and the bars. Two extreme cases were assumed: no friction, and no relative movement between the specimen and the bars at their interfaces.

### 7.2.3 Computation Results

The computation results for the case of no friction are given first. Though the conventional notation was used in modelling and program UFCONC, in order to compare computed quantities with test records, compressive stress, longitudinal compressive strain and hoop tensile strain are still taken as positive as in Chapters 2 to 5. Strains of the specimen were calculated from the displacement output of the computation by UFCONC. The average longitudinal strain over the whole specimen was calculated from the output longitudinal displacements of node O and node O', designated as  $\epsilon'$ . The average longitudinal strain at the midpoint of the specimen length on the lateral surface, designated as  $\epsilon'_s$ , was calculated from the longitudinal displacements of node A and node B. Being the midpoints of an edge of element 9 and element 12, respectively, as shown in Figure 7.3, node A and node B are 0.5 inch apart, which is about the length of the strain gage used to record  $\epsilon_s$ . The

lateral strain at the midpoint of the specimen length,  $\epsilon'_\theta$  was calculated from the radial displacement of the node halfway between A and B. All  $\epsilon'_1$ ,  $\epsilon'_2$  and  $\epsilon'_\theta$  are shown in Figures 7.4 to 7.7 to be compared with  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_\theta$  for specimens W24, W20, WT3 and W28. Computational longitudinal strains are most close to the experimental data just before the maximum stress. So far as the lateral strain is concerned, the computational and experimental data for W28 coincide dramatically well. For lower-speed impact tests, where lower dynamic compressive strengths were observed,  $\epsilon'_\theta$  deviates more from  $\epsilon_\theta$  than that of specimen W28, which was tested with the highest impact speed, 600 psi. Though the record of  $\epsilon_\theta$  for WT3 was thought bad so that it was not employed in the one-dimensional analysis (Chapters 3 and 4), the discrepancy between  $\epsilon'_\theta$  and  $\epsilon_\theta$  may be also tolerable. The coincidence of the computational and experimental strain data indicates that the proposed model well describes the unconfined dynamic tests up to the maximum stress. The associative flow rule for viscoplastic strain rates is a satisfactory assumption for the WES high-strength concrete, at least in the test environment up to the maximum stress on a first loading.

Figures 7.8 to 7.15 are computation-output stresses at six Gauss points in elements 7, 8 and 9 for specimens W20 and W28. All the traces in these figures are marked with characters a, b, c, d, e and f, which tell at which Gauss

point the stress is. All these points have the same longitudinal location, that is,  $z = 1.394$  inch. The radial coordinate is 0.106 inch, 0.394 inch, 0.606 inch, 0.894 inch, 1.106 inch and 1.394 inch for a, b, c, d, e and f, respectively. Compressive stresses are taken as positive in these stress figures, and, for shear stress  $\sigma_{xz}$  on a section with the positive  $z$ -direction as its outward normal, the shear stress is taken positive if it directs along the negative  $r$ -direction. Time starting points in the stress diagrams for W20 are corresponding to 71.5 microsecond in Figure 7.5, and time starting points in the stress figures for W28 are corresponding to 60 microsecond in Figure 7.7. As seen in Figures 7.8 and 7.12, the higher the impact velocity used in the test, the higher the radial normal stress  $\sigma_{xx}$ . However, even for W28, the radial normal stress peak is only five percent of the maximum longitudinal normal stress. This confirms the conclusion made in Chapter 3 that no serious radial forces were caused by the lateral inertia. As seen in the figures,  $\sigma_{xz}$  is very small at e and f, because of the free-traction boundary condition on the lateral surface of the specimen, but  $\sigma_{\theta\theta}$  at e and f are relatively large. The maximum values of these tensile normal stresses for W28 are larger than those for W20, while the longest intervals of tensile  $\sigma_{xx}$  and  $\sigma_{\theta\theta}$  in W20 are about 30 microseconds, longer than those of about 20 microseconds in W28.

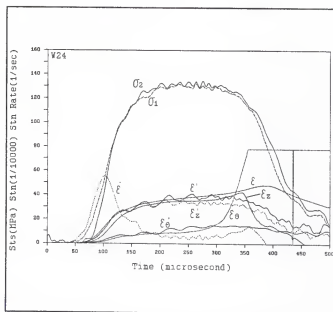


Figure 7.4 Comparison of Computational and Experimental Strains for W24

Note:  $\epsilon$  is the average strain of the specimen from the SHPB records.

$\epsilon_z$  and  $\epsilon_\theta$  are, respectively, the local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface, measured by the gages mounted on the specimen.

$\epsilon'$  is the computed average strain over the specimen length along the centerline.

$\epsilon'_z$  and  $\epsilon'_\theta$  are, respectively, the computed local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface.

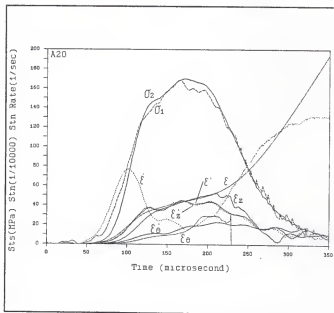


Figure 7.5 Comparison of Computational and Experimental Strains for W20

Note:  $\epsilon$  is the average strain of the specimen from the SHPB records.

$\epsilon_z$  and  $\epsilon_\theta$  are, respectively, the local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface, measured by the gages mounted on the specimen.

$\epsilon'$  is the computed average strain over the specimen length along the centerline.

$\epsilon'_z$  and  $\epsilon'_\theta$  are, respectively, the computed local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface.

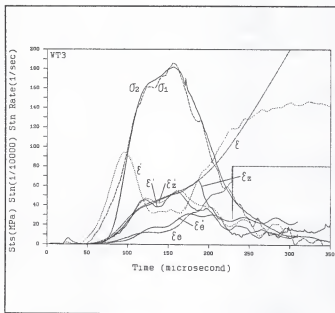


Figure 7.6 Comparison of Computational and Experimental Strains for WT3

Note:  $\epsilon$  is the average strain of the specimen from the SHPB records.

$\epsilon_z$  and  $\epsilon_\theta$  are, respectively, the local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface, measured by the gages mounted on the specimen.

$\epsilon'$  is the computed average strain over the specimen length along the centerline.

$\epsilon'_z$  and  $\epsilon'_\theta$  are, respectively, the computed local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface.

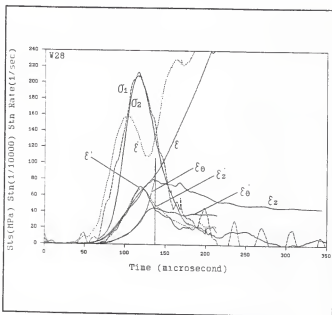


Figure 7.7 Comparison of Computational and Experimental Strains for W28

Note:  $\epsilon$  is the average strain of the specimen from the SHPB records.

$\epsilon_x$  and  $\epsilon_\theta$  are, respectively, the local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface, measured by the gages mounted on the specimen.

$\epsilon'$  is the computed average strain over the specimen length along the centerline.

$\epsilon'_x$  and  $\epsilon'_\theta$  are, respectively, the computed local longitudinal and hoop strain at the midpoint of the specimen length on the lateral surface.



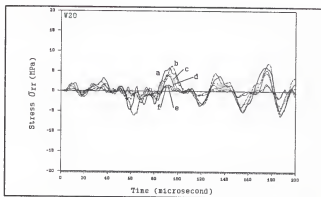


Figure 7.8 Computational Radial Normal Stresses for W20

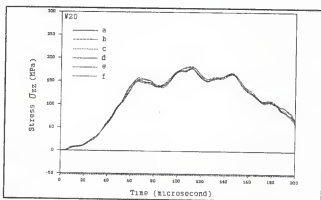


Figure 7.9 Computational Longitudinal Normal Stresses for W20

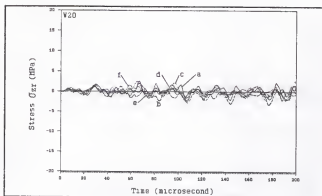


Figure 7.10 Computational Shear Stresses for W20

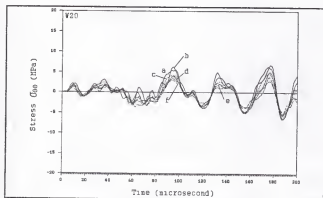


Figure 7.11 Computational Hoop Normal Stresses for W20

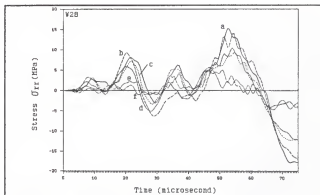


Figure 7.12 Computational Radial Normal Stresses for W28

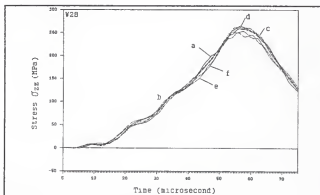


Figure 7.13 Computational Longitudinal Normal Stresses for W28

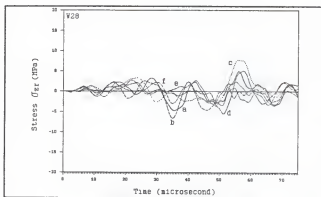


Figure 7.14 Computational Shear Stresses for W28

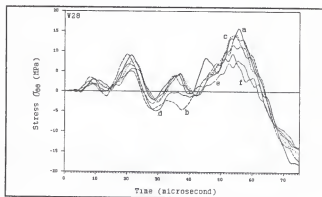


Figure 7.15 Computational Hoop Normal Stresses for W28

There has been some question whether friction between the specimen end faces and the bars provides enough lateral constraint on the specimen to enhance the longitudinal strength of the concrete specimen. The inelastic-strain-rate dependence summarized in Chapters 3, 4 and 7 was based on the assumption that the specimen being tested was uniaxially loaded. It was necessary to perform computation analysis with friction to check reasonability of the no-friction assumption used in analysis before.

In the analysis with friction, for simplicity, the bars are assumed to be in uniaxial stress with both lateral inertia and the radial friction on the end ignored, so that their radial movement is controlled only by Poisson's ratio, and the friction is assumed such that the radial displacements at each node on the specimen end surfaces are specified identical to the assumed radial displacements of the point on the bar in contact with the node. It is obviously an extreme case of friction because the concrete tends to expand more laterally than the steel bars under the same axial stress. The stresses and strains computed in this extreme friction case were calculated for all the same points and same components as in the no-friction case. Local average longitudinal strain  $\epsilon'_x$  and hoop strain  $\epsilon'_\theta$  computed for W28 are presented in Figure 7.15, which are close to the test records  $\epsilon_x$  and  $\epsilon_\theta$  as shown, but  $\epsilon'_x$  computed at the specimen centerline is rather larger than the  $\epsilon_x$  that

was acquired in the test. Indeed, friction makes the average longitudinal strain over the specimen length vary along the radius. Figure 7.16 shows the average longitudinal strain over the specimen length at different radii 0, 0.5, 1.0 and 1.5 inch marked with  $\epsilon^I$ ,  $\epsilon^{II}$ ,  $\epsilon^{III}$  and  $\epsilon^{IV}$  respectively as well as  $\epsilon'_s$  and the two test recorded strains. Stress distributions are shown in Figures 7.17 to 7.18, where the starting time zero corresponds to 60 microsecond in Figures 15 and 16. Friction causes compressive  $\sigma_{xx}$  to be a bit higher than that in the no-friction case, but also causes  $\sigma_{xx}$  in the center of the specimen to be larger than that measured on the SHPB. However, friction also causes higher and longer-lasting tensile  $\sigma_{xx}$  and  $\sigma_{\theta\theta}$ , which may be the cause of longitudinal cracks leading to longitudinal strips of the fractured specimen remains. In consistency with the varying longitudinal strain over the radius, stress  $\sigma_{xx}$  distribution over the radius is not uniform.

In addition, computation with a constant friction coefficient was executed. Since the concrete specimens were well ground and lubricated to contact the bars in place before test, the friction coefficient was assumed to be 0.15 (Crandall and Dahl, 1972). The friction was assumed always towards the specimen centerline for simplicity, though the radial movements of the specimen end surfaces were affected by the particle movements inside the specimen and therefore

may not always and everywhere be directed outward relative to the bar end surfaces. The computation on W28 with the proposed model and the above friction assumptions was performed. Figure 7.22 compares the radial displacements of the edge of the incident and transmitter interfaces of the specimen,  $d_s^i$  and  $d_s^t$ , as well as the radial displacements of the lateral surface of the incident bar and transmitter bar,  $d^i$  and  $d^t$ , calculated from longitudinal linear strains and Poisson's ratio only. All the displacements in the positive  $r$ -direction are expressed as positive and the figure starts at the same time as Figure 7.7 does. In the beginning part,  $d^i$  goes up faster than  $d_s^i$  so that the friction should pull the specimen surfaces out rather than push it. In another part, where  $d_s^i$  goes down while  $d^i$  goes up, the friction direction could not be toward the inside either. Nevertheless, the computation showed that the friction direction assumption was mostly all right before unloading. Computation outputs from the two friction-assumed cases for all the stress and strain quantities are very close. Figure 7.23 gives the computed strains  $\epsilon'$ ,  $\epsilon'_z$  and  $\epsilon'_\theta$  for the extreme friction case, and  $\epsilon'_{(0.15)}$ ,  $\epsilon'_{z(0.15)}$  and  $\epsilon'_{\theta(0.15)}$  for the case of friction coefficient of 0.15. The local longitudinal strains and hoop strains from the two cases all coincide well with the test data. The average longitudinal strains over the specimen length taken at the specimen centerline,  $\epsilon'$  and  $\epsilon'_{(0.15)}$ , deviate from the tested strains though they

are close to each other. Stress distributions from the two cases are also so similar that figures for the stress distributions for the case of friction coefficient of 0.15 are superfluous to show here.

According to the computation in the no-friction assumed case and the two friction-assumed cases, friction apparently causes inconsistency of longitudinal normal stress distribution over the cross section in the specimen center. A question whether the normal-stress distribution is uniform at the specimen ends arises and turns consideration back to an assumption which was made in the simple stress wave theory and maintained in the stress wave dispersion correction (Chapter 2), that is, that the longitudinal normal stress distributes evenly across the section in the bars. So far as the calculated displacement distribution is concerned, it is quite uniform over the specimen cross section in the no-friction case, so that average strains are quite uniform too, but friction breaks this uniformity.



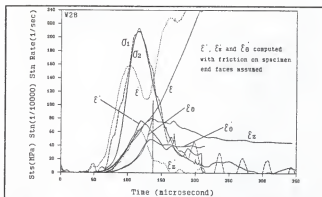


Figure 7.16 Comparison of Computational and Experimental Strains for W28 with Friction Assumed

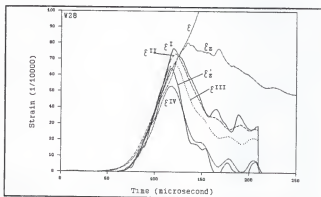


Figure 7.17 Average Longitudinal Strains at Different Radii for W28 with Friction Assumed

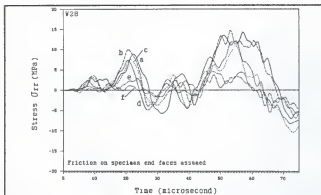


Figure 7.18 Computational Radial Normal Stresses for W28 with Friction Assumed

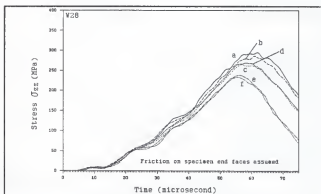


Figure 7.19 Computational Longitudinal Normal Stresses for W28 with Friction Assumed

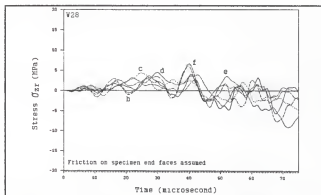


Figure 7.20 Computational Shear Stresses for W28 with Friction Assumed

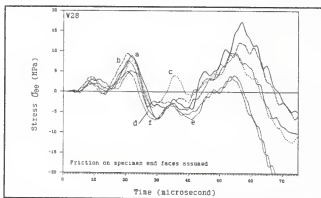


Figure 7.21 Computational Hoop Normal Stresses for W28 with Friction Assumed

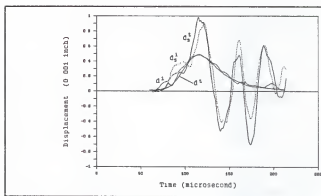


Figure 7.22 Displacements of the Edge of the Specimen Surfaces and the Lateral surface of the Bars in Test W28 with Friction Coefficient of 0.15 Assumed

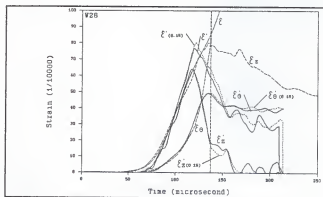


Figure 7.23 Comparison of Strains Computed in Two Friction-Assumed Cases and for W28

## CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

### 8.1 Introduction

This study investigated the behavior of concrete under dynamic compressive loading. A cycle of macroscopic investigation - testing, modeling, computation and then comparison of computation output with test records was completed. In addition, cracks in dynamically-damaged specimens were observed. All the dynamic tests were performed on the SHPB system under uniaxial compressive loading. Conclusions and recommendations are summarized as follows.

### 8.2 Conclusions

1. The 3-inch-diameter Split Hopkinson's Pressure Bar (SHPB) system is a useful tool for determining the dynamic stress-strain curve of concrete at strain rates from about 5/sec to 230/sec. The dispersion correction of the stress wave along the bars leads to much closer agreement of the two interface stresses.

2. Unconfined specimens of mortar and five kinds of high-strength concretes were tested. Each of them showed strain-rate dependence of the compressive strength. The mortar (tested in a smaller SHPB system) showed an apparent linear dependence of the compressive strength on the strain rate over the strain-rate range from 50 to 800/sec, while two kinds of the high-strength limestone-aggregate concrete prepared by the Waterways Experimental Station (WES) and SRI International could be described by either a semilogarithmic or a power-law dependence over the strain-rate range from 5 to 230/sec. An apparent rate dependence up to twice the static strength is observed for both concrete and mortar.

3. A rate-dependent model was suggested to describe the behavior of concrete specimens under uniaxial compressive loading, by which the stress in loading simply depends on inelastic strain rate. A cumulative criterion of fracture initiation was suggested following a definition of a damage parameter  $K$ , the time integral of an amount by which the compressive stress pulse exceeds the compressive yield stress. When  $K$  is increased up to a critical value  $K^c$ , the maximum stress of the specimen occurs. This criterion gives the limit for the rate-dependent model.

4. The collar method was successful in arresting the tests, so that intact damaged specimens were recovered for micrographic examination of the crack distributions, and it was possible to deduce the maximum strain level reached in

these tests. The sectioning, infiltration, and polishing did not introduce any significant additional macrocracking in the specimens.

5. It appears that a substantial increase in crack surface per unit volume occurs at a threshold strain near the strain at peak stress of the no-collar tests. The threshold strains of the dynamic and static tests are about the same. The crack surface per unit volume is considerably higher in the dynamic tests than in the static tests at the same level of maximum strain. This suggests that in static tests there tends to be on the average more sliding on each crack than in dynamic tests.

6. A rate-independent plastic model for concrete in the three-dimensional stress case was constructed with other researchers' static-test data. An over-stress type of elastoviscoplastic model in the three-dimensional stress case was then proposed incorporating some features of the previous plastic model and fitted with experimental data on rate dependence obtained in this study. Program UFGONC was compiled for application of the elastoviscoplastic model in the plane stress, plane strain or axisymmetric cases and has been proved workable for the analysis of the axisymmetric SHPB specimen. The explicit, central difference scheme is used for direct time integration in the finite element technique. This program is able to perform stress and displacement analysis under time-varying loads.

7. SHPB tests on WES high-strength concrete specimens were analyzed by UFGCONC. When no friction was assumed between the specimen and the incident and transmitter bars of the SHPB system, and the measured axial stresses of the two end faces of the specimen (assumed uniform over the end faces) were input as loads, UFGCONC yielded the time histories of the specimen average longitudinal strain, the local longitudinal strain at the midlength on the lateral surface, and the hoop strain at the midlength, all of which coincided well with experimental data. This result shows that the proposed elastoviscoplastic model can, at least in the uniaxial compressive case, predict deformation of the concrete under prescribed rapid loading. The prediction of the hoop strain indicates that the associative flow rule applied to the Willam-Warnke surface is effective.

8. When friction was assumed between the specimen and the bars, computed local longitudinal strain and hoop strain, both at the midlength on the lateral surface, coincided well with the recorded data acquired by the strain gages which were mounted on the specimen. With friction assumed, computed average longitudinal linear strains over the whole specimen length were not uniform, but the SHPB recorded specimen strain lay in the range of these computed strains. No evidence violates the proposed constitutive model. According to computation results, friction causes higher radial normal stress in the specimen center but



longitudinal normal stress in the center part is also higher than either that computed with no friction assumed or the stress that was recorded by the SHPB.

9. Analysis based on the data acquired by the strain gage mounted on the specimen laterally showed no significant radial inertia pressure during the SHPB test, which was contrary to suggestions that radial confinement was the cause of the dynamic strength increase.

10. Before the maximum stress was reached in the SHPB tests, hoop normal stress and radial normal stress once became tensile, and friction on the specimen end surfaces enhanced these tensile stresses. These tensile stresses may have initiated and developed axially-oriented cracks and led to specimen fracture in unconfined tests.

### 8.3 Recommendations

1. Test data are not enough yet to completely verify the proposed three-dimensional constitutive model. So far the strain-hardening cap is assumed rate insensitive because not enough dynamic test data are available to describe the cap movement. In the unconfined SHPB tests, it took a rather short time for loading to go from the initial cap through the Willam-Warnke surface so that, even if the cap should be rate sensitive, the neglect of the rate sensitivity could not have brought about considerable

errors. Unconfined SHPB tests provided a loading path along which the angle of similarity remained  $\pi/3$ . Even though the lateral inertia and friction were included in the computation, the loading path for any point in the specimen could not deviate much from the plane of the angle of similarity equal to  $\pi/3$ . Therefore, dynamic experiments to make different loading paths should be designed and carried out.

2. The program UFCONC could be applied to the dynamic tests under various loading paths to check the validity of the constitutive model. For example, it could be used to analyze confined tests like those of Malvern et al. (1989), where lateral hydrostatic pressure was applied by a surrounding pressure chamber before the dynamic axial SHPB loading occurred. Although the lateral confining pressures were small compared to the axial strength, they caused significant increase in the dynamic strength, and large deformations occurred without failure. Future tests of this type should also include surface strain gage measurements and lubricated specimen end surfaces.

3. An evolution law based on crack or other damage accumulation is needed to determine the time (or the critical strain or stress condition) beyond which the model does not describe the behavior. For the analysis of the SHPB specimen where the two experimental end-face stress histories were available, that time was the time when the

maximum stress occurred, but for applications without such information a predictive criterion is needed.

4. Specimens near the threshold strain in crack observation are especially interesting. A closely spaced series to precisely map out the threshold region would be useful, and it should be possible to take a closer look at the process zones around the developing cracks, possibly using *scanning electron microscope* methods. More sections, on some specimens at least, should be taken to improve the statistics and reduce any orientation bias. Stress measurements in the dynamic tests might be improved by more accurate alignment of the collar and use of strain gages on the elastic collar, which should make it possible to determine the load carried by the collar, so that the load carried by the concrete could be calculated. The crack identification was a tedious process because of the prominence of other micrographic features. It should be possible to facilitate the procedure by adding a fluorescent dye to the furfuryl alcohol before infiltration into the cracks and using ultraviolet illumination, so that only the cracks will show up in the micrograph. It might then be possible to automate the crack counting. Fluorescent-dye techniques have been reported previously in examination of very thin slices to observe porosity and microcracking; see

Gardner (1980) and Knab et al. (1986). They should also be useful to observe the macrocracks in thicker slices.

5. If it were possible to determine experimentally the radial distribution of the axial stress or displacement on the end faces of the SHPB specimen, then they could be input to program UFCNC to get a better check on the model.

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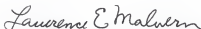
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## BIOGRAPHICAL SKETCH

Tianxi Tang was born in Wuxi, Jiangsu Province, China, on July 14, 1943. After graduating from a high school in Wuxi, he left his hometown in fall 1960 for a Chemical Engineering Institute in Nanjing, Jiangsu, and then studied there for three years. He entered Shanghai Jiao Tong University in fall 1963 and met his future wife Yaoqi Pang there. He graduated from Shanghai Jiao Tong University in December 1968. From 1969 on, he worked in Haozhitang Coal-Mine Machinery Factory in Guiyang, Guizhou Province, as an assistant engineer until he transferred to Hefei, Anhui Province, to continue his industry career in Hefei Hydraulic Press Works in 1974. In 1978 he started his new job at the University of Science and Technology of China, Hefei, Anhui Province, as a lecturer. Tianxi Tang came to the United States in June 1982 to perform research at the University of Florida, Gainesville, Florida, as a visiting scholar. He was admitted to be a graduate student in the University of Florida in summer 1985 and since then he has worked towards his doctorate degree with a major of engineering mechanics.

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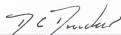
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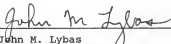


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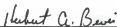
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